Exercises June 11th 2004, Optimal Control of Economic Systems

1. Consider the scalar system

   $$\dot{x}(t) = x(t)u(t), \qquad x(0) = 1$$

   with cost criterion $J(x_0, u(\cdot)) = 2x(T) + \int_0^T x^2(t) + u^2(t)dt$.

   (a) Determine the minimal cost and the optimal input u.
   Hint : Try $V(t, x) = q(x)$ as a candidate value function in the Hamiltion-Jacobi-Bellman equation

   (b) Calculate the optimal state trajectory for $T = 2$.

   (c) Calculate the "optimal" state trajectory for $x(0) = -1$ and $T = 2$. Conclude that the corresponding input u is not admissible.

2. Exercise 4.2

3. Exercise 4.3

4. Consider the following system

   $$\begin{aligned} \dot{x}_1(t) &= u(t)x_1(t) \\ \dot{x}_2(t) &= (1 - u(t))x_1(t) \end{aligned} \qquad (1)$$

   We assume that $x_1(0) = x_{10} \geq 0$, $x_2(0) = x_{20}$ and $u(t) \in [0, 1]$. Furthermore, we assume that u is piecewise continuous.

   With this system we have the cost criterion given by

   $$J_1(u(\cdot)) = \int_0^T x_2(t)dt. \qquad (2)$$

   The aim is to maximize this criterion.

   (a) Give the Hamiltonian equations and determine the candidate solution to the optimal control problem.

   Let us now assume that $x_{10} = 0$.

   (b) What is the maximal cost?

   (c) Give all solutions to the optimal control problem.