The Polyphase Implementation of FIR Filters

Implementation of Digital Signal Processing

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OUTLINE

- Side topic, project background: GFSK receiver + testbench
- Downconversion and downsampling
- Polyphase implementation
- Upsampling

LITERATURE

- To study:

- Optional texts for in-depth information:

DESIGN EXAMPLE: GFSK RECEIVER

- What is GFSK?
  - Gaussian frequency shift keying
  - Method for digital transmission based on frequency modulation (FM).
  - To transmit a 1 carrier frequency is slightly increased and to transmit a 0 the frequency is slightly decreased (or vice versa).
  - The transition steps are smoothed by a Gaussian filter.
  - Found in many standards: Bluetooth, DECT, Wavenis, …
  - Proposed version uses parameters not related to any standard.
**GFSK RECEIVER DESIGN APPROACH**

- Model entire system: transmitter, receiver, and a channel adding noise (AWGN).
- Leave out analog circuitry for upconversion to RF and downconversion back to IF.
- Use Synopsys System Studio to set up testbench.
- The testbench computes bit error rates (BERs) for different signal-to-noise ratios (SNRs).
- Goal is to preserve BER performance when designing hardware.

**GFSK: MODULATION IN FORMULAE**

- The modulated signal: 
  \[ s(t) = A \cos(\omega_{IF} t + \phi(t)) \]
- where:
  - \( A \) is the constant amplitude
  - \( \omega_{IF} \) is the *intermediate frequency* (acts as carrier frequency)
  - \( \phi(t) \) is the phase deviation, derived from the bit stream
- The phase deviation:
  \[ \phi(t) = h \pi \int_{-\infty}^{t} \sum_{i} a_i g(\tau - iT) \, d\tau \]
- where:
  - \( h \) is the modulation index
  - \( g(t) \) is a Gaussian-filtered square wave
  - \( a_i \) is 1 for a transmitted 1 and -1 for a transmitted 0.
DEMODULATOR BLOCK DIAGRAM

The 16 samples per transmitted bit are first reduced to 4 and later back to 1.

SPECTRUM

• Before downconversion:

• After mixing:

• After low-pass filtering:

One could use a lower sampling frequency at this stage

DOWNSampling

• Operation in DSP where 1 out of N samples is kept (the other N-1 are thrown away).

• Sometimes also called decimation.

• In example application, this would amount to:

Not efficient to compute all LPF outputs and keep only 1 in N.

DOWNSampling IN frequency Domain

• Spectrum before downsampling:

• Spectrum after downsampling with factor 6:
NOBLE IDENTITY FOR DOWNSAMPLING

- The context is a filter followed by a downsampler:

\[ H(z^N) \downarrow \]

\[ x[n] \rightarrow y[n] \rightarrow y[Nn] \]

- Interpretation: filtering and downsampling can be swapped provided that delays in filter are N-fold (normally not true!).

The noble identity holds for any filter, not just for FIR filters!

POLYPHASE FILTERING EXAMPLE (1)

- Consider Kth-order FIR filter with transfer function \(H\) given by coefficients \(b\):

\[ y[n] = \sum_{k=0}^{K} b[k] \cdot x[n-k] \]

- Example: downsampling by 3 after filtering, how to implement efficiently?

\[ x[n] \rightarrow H(z) \rightarrow |3| \rightarrow y[3n] \]

POLYPHASE FILTERING EXAMPLE (3)

- Consider outputs after downsampling and rewrite by grouping coefficients with offsets of 3:

\[ y[3n] = \sum_{k=0}^{K} b[k] \cdot x[3n-k] \]

\[ = \sum_{k=0}^{K/3} b[3k] \cdot x[3(n-k)] + \sum_{k=0}^{K/3} b[3k+1] \cdot x[3(n-k) - 1] + \sum_{k=0}^{K/3} b[3k+2] \cdot x[3(n-k) - 2] \]

- Graphical representation of rewriting:
**POLYPHASE FILTERING EXAMPLE (4)**

- Now the noble identity can be applied to the three subfilters:

\[
x[n] \xrightarrow{\downarrow 3} H_0(z) \xrightarrow{\downarrow 3} H_1(z) \xrightarrow{\downarrow 3} H_2(z) \rightarrow y[3n]
\]

Original sample rate \( \rightarrow \) Reduced sample rate

**CHALLENGE**

- Going back to the GFSK case …
- The low-pass filter can be implemented in polyphase.
- What about the downconversion preceding the LPF? Can it also be implemented in the low-sample-rate side?

**FILTER BANKS (1)**

- Separate signal into adjacent frequency bands, by means of band-pass filters
- Each band has limited bandwidth and can therefore reduce its sample rate, polyphase solution can be applied!

**FILTER BANKS (2)**

- Signal reconstruction after subband processing requires **upsampling**.
- Filtering after upsampling is required to avoid aliasing.
UPSAMPLING

- Operation in DSP where N samples are produced for each input (N-1 zeros are inserted between original samples)

\[ \begin{array}{c}
1 \\
\uparrow \\
N \\
\downarrow \\
N
\end{array} \]

- Sometimes also called interpolation.

UPSAMPLING IN FREQUENCY DOMAIN

- Spectrum before upsampling:

\[ \omega_s \]

- Spectrum after upsampling with factor 6:

\[ \frac{6\omega_s}{2} \]

- Low-pass filtering is necessary to remove aliases.
- It is not efficient to feed zeros to filter.

THE NOBLE IDENT. FOR UPSAMPLING

- The context is an upsampler followed by a filter:

\[ x[Mn] \xrightarrow{M} x[n] \xrightarrow{H(z^M)} y[n] \]

The noble identity holds for any filter, not just for FIR filters!

POLYPHASE FILTERING EXAMPLE (1)

- Consider K\textsuperscript{th}-order FIR filter with transfer function \( H \) given by coefficients \( b \):

\[ y[n] = \sum_{k=0}^{K} b[k] \cdot x[n-k] \]

- Example: upsampling by 3 followed by filtering, how to implement efficiently?

\[ x[Mn] \xrightarrow{3} x[n] \xrightarrow{H(z)} y[n] \]
POLYPHASE FILTERING EXAMPLE (2)

• Start with definition, and group by coefficient index:

\[ y[n] = \sum_{k=0}^{K} b[k] \cdot x[n - k] \]

\[ = \sum_{k=0}^{K_0} b[3k] \cdot x[n - 3k] + \sum_{k=0}^{K_1} b[3k + 1] \cdot x[n - 3k - 1] + \sum_{k=0}^{K_2} b[3k + 2] \cdot x[n - 3k - 2] \]

Depending on \( n \), only one out of three groups will be unequal to zero!

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POLYPHASE FILTERING EXAMPLE (3)

• Now consider outputs with different offsets separately and keep only those inputs unequal to zero.

• The result consists of three sequences that are filtered versions of the signal before upsampling.

\[ y[3n] = \sum_{k=0}^{K_0} b[3k] \cdot x[3(n - k)] \]

\[ H_0(z^3) \]

\[ y[3n + 1] = \sum_{k=0}^{K_1} b[3k + 1] \cdot x[3(n - k)] \]

\[ H_1(z^3) \]

\[ y[3n + 2] = \sum_{k=0}^{K_2} b[3k + 2] \cdot x[3(n - k)] \]

\[ H_2(z^3) \]

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POLYPHASE FILTERING EXAMPLE (4)

• The previous equations represent:

\[ x[3n] \]

\[ \uparrow 3 \]

\[ x[n] \]

\[ H_0(z^3) \]

\[ y[3n] \]

\[ H_1(z^3) \]

\[ y[3n + 1] \]

\[ H_2(z^3) \]

\[ y[3n + 2] \]

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POLYPHASE FILTERING EXAMPLE (5)

• After applying the noble identity for upsampling:

\[ x[3n] \]

\[ H_0(z) \]

\[ y[3n] \]

\[ H_1(z) \]

\[ y[3n + 1] \]

\[ H_2(z) \]

\[ y[3n + 2] \]

\[ y[n] \]

• Note: the upsample nodes have been left out as they produce zeros when the switch is not using their outputs.