The Polyphase Implementation of FIR Filters

Implementation of Digital Signal Processing

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OUTLINE

• Downconversion and downsampling
• Polyphase implementation
• Upsampling

LITERATURE

• To study:

• Optional texts for in-depth information:

DEMODULATOR BLOCK DIAGRAM

The 16 samples per transmitted bit are first reduced to 4 and later back to 1.
SPECTRUM

• Before downconversion:

0 \quad \omega_{IF} \quad \frac{\omega_s}{2}

• After mixing:

0 \quad \omega_{IF} \quad 2\omega_{IF} \quad \frac{\omega_s}{2}

• After low-pass filtering:

One could use a lower sampling frequency at this stage

0 \quad \omega_{IF} \quad 2\omega_{IF} \quad \frac{\omega_s}{2}

DOWNSAMPLING

• Operation in DSP where 1 out of N samples is kept (the other N-1 are thrown away).

\[ \frac{1}{N} \]

• Sometimes also called *decimation*.

In example application, this would amount to:

Original sample rate \quad \rightarrow \quad \text{Reduced sample rate}

Not efficient to compute all LPF outputs and keep only 1 in N.

DOWNSAMPLING IN FREQUENCY DOMAIN

• Spectrum before downsampling:

0 \quad \omega_{IF} \quad 2\omega_{IF} \quad \frac{\omega_s}{2}

• Spectrum after downsampling with factor 6:

\[ \frac{\omega_s}{12} \quad \frac{\omega_s}{2} \]

The noble identity holds for any filter, not just for FIR filters!
**POLYPHASE FILTERING EXAMPLE (1)**

- Consider $K$th-order FIR filter with transfer function $H$ given by coefficients $b$:

  $$y[n] = \sum_{k=0}^{K} b[k] \cdot x[n-k]$$

- Example: downsampling by 3 after filtering, how to implement efficiently?

  ![Diagram](image1)

**POLYPHASE FILTERING EXAMPLE (3)**

- Graphical representation of rewriting:

  ![Diagram](image2)

**POLYPHASE FILTERING EXAMPLE (4)**

- Now the noble identity can be applied to the three subfilters:

  ![Diagram](image3)
FILTER BANKS (1)

- Separate signal into adjacent frequency bands, by means of band-pass filters
- Each band has limited bandwidth and can therefore reduce its sample rate, polyphase solution can be applied!

Analysis filter bank, used e.g. in subband coding for audio.

FILTER BANKS (2)

- Signal reconstruction after subband processing requires upsampling.
- Filtering after upsampling is required to avoid aliasing.

UPSAMPLING

- Operation in DSP where N samples are produced for each input (N-1 zeros are inserted between original samples)
- Sometimes also called interpolation.

UPSAMPLING IN FREQUENCY DOMAIN

- Spectrum before upsampling:

  \[ 0 \quad \frac{\omega_s}{2} \]

- Spectrum after upsampling with factor 6:

  \[ 0 \quad \frac{6\omega_s}{2} \]

- Low-pass filtering is necessary to remove aliases.
- It is not efficient to feed zeros to filter.
THE NOBLE IDENT. FOR UPSAMPLING

- The context is an upsampler followed by a filter:

\[ x[Mn] \rightarrow \uparrow M \rightarrow H(z^M) \rightarrow y[n] \]

- Interpretation: filtering and upsampling can be swapped provided that delays in filter are M-fold (normally not true!).

POLYPHASE FILTERING EXAMPLE (1)

- Consider \( K \)-th order FIR filter with transfer function \( H \) given by coefficients \( b \):

\[ y[n] = \sum_{k=0}^{K} b[k] \cdot x[n-k] \]

- Example: upsampling by 3 followed by filtering, how to implement efficiently?

\[ x[Mn] \uparrow 3 \rightarrow x[n] \rightarrow H(z) \rightarrow y[n] \]

POLYPHASE FILTERING EXAMPLE (2)

- Start with definition, and group by coefficient index:

\[ y[n] = \sum_{k=0}^{K} b[k] \cdot x[n-k] \]

\[ = \sum_{k=0}^{K_0} b[3k] \cdot x[n-3k] + \]

\[ + \sum_{k=0}^{K_1} b[3k+1] \cdot x[n-3k-1] + \]

\[ + \sum_{k=0}^{K_2} b[3k+2] \cdot x[n-3k-2] \]

Depending on \( n \), only one out of three groups will be unequal to zero!

POLYPHASE FILTERING EXAMPLE (3)

- Now consider outputs with different offsets separately and keep only those inputs unequal to zero.

- The result consists of three sequences that are filtered versions of the signal before upsampling.

\[ y[3n] = \sum_{k=0}^{K_0} b[3k] \cdot x[3(n-k)] \]

\[ y[3n+1] = \sum_{k=0}^{K_1} b[3k+1] \cdot x[3(n-k)] \]

\[ y[3n+2] = \sum_{k=0}^{K_2} b[3k+2] \cdot x[3(n-k)] \]

\[ H_0(z^3) \]

\[ H_1(z^3) \]

\[ H_2(z^3) \]
POLYPHASE FILTERING EXAMPLE (4)

- The previous equations represent:

\[ x[3n] \uparrow 3 \rightarrow x[n] \rightarrow H_0(z^3) \rightarrow y[3n] \rightarrow H_1(z^3) \rightarrow y[3n+1] \rightarrow H_2(z^3) \rightarrow y[3n+2] \rightarrow y[n] \]

POLYPHASE FILTERING EXAMPLE (5)

- After applying the noble identity for upsampling:

\[ x[3n] \rightarrow H_0(z) \rightarrow y[3n] \rightarrow H_1(z) \rightarrow y[3n+1] \rightarrow H_2(z) \rightarrow y[3n+2] \rightarrow y[n] \]

- Note: the upsample nodes have been left out as they produce zeros when the switch is not using their outputs.