Transport Capacity of Wireless Networks: Benefits from Multi-Access Computation Coding

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Abstract

We consider the effect on the transport capacity of wireless networks of different physical layer coding mechanisms. We compare the performance of traditional channel coding techniques, turning the wireless network in reliable point-to-point channels, with multi-access computation coding, in which nodes receive functions of messages transmitted by different neighbours. In both cases, network coding is used on higher layers. For one-dimensional networks, the benefit in transport capacity of computation-coding over point-to-point channels is a factor of 2; for two-dimensional networks, we show it to be at least 2.5.

1 Introduction

The benefits of network coding were first demonstrated for multicast problems in networks of point-to-point channels [1]. More recently, it was shown that in wireless networks, that inherently do not consist of point-to-point channels, there is a great potential in applying network coding for multiple unicast problems [2]. Some of the potential benefits are increased throughput [2] and reduced energy consumption [3, 4]. The work in [2–4] is based on exploiting the broadcast effect of the wireless medium. By allowing multiple nodes to receive the same message, coding opportunities arise.

Of course, nodes also receive signals from multiple other nodes simultaneously. More recently, it was shown independently by several authors that this can in fact also be exploited in combination with linear network coding to gain in throughput [5–7]. The work in [5] and [6] focuses mostly on recovering the sum of messages, based on uncoded transmissions at the transmitters. A more general approach is taken in [7], using results from [8], giving upper and lower bounds on the rate at which one can reliably communicate a function of several messages across a multi-access channel. The above techniques are known in the literature under different names, for instance physical-layer network coding, analog network coding or multi-access computation coding.

In this paper we take the following approach. Channel coding techniques are used to provide a means to reliably communicate over the noisy wireless medium. Network coding is then used to perform operations on reliably received data. Traditionally, channel coding is done in such a way, that the wireless medium is turned into point-to-point channels. There are, however, many other coding techniques that can be used to reliably communicate. An overview of some of these techniques is presented in [9]. In this work, we analyze the difference in network capacity arising from two of these techniques, where in both cases we allow network coding to be performed on higher

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layers. We compare 1) point-to-point channels (PP) and 2) broadcasting together with multi-access computation coding (CC).

The traffic pattern that we consider is multiple unicast and the capacity measure of interest is the transport capacity of a network, *i.e.*, the maximum of the weighted sum of the throughputs per session, where the weight is the distance between source and receiver and the maximum is over all multiple unicast configurations. We show that for a line network, the ratio of transport capacity under CC and under PP is 2, a result previously obtained in [5]. Using the proof techniques developed for the line network we show that on the hexagonal lattice, the ratio is at least 2.5 and at most 6.

In Section 2 we formulate our model. Some of the characteristics of the model are captured in Section 3. In Section 4 we analyze a line network and in Section 5 the hexagonal lattice. In Section 6 we provide a discussion of our results.

2 Model and Notation

We model a wireless network as an undirected graph $G(V,E)$, where V is the set of nodes and $E \subseteq V \times V$ are the edges, which represent the interaction between nodes. For notational convencience, we consider directed edges, *i.e.*, $(u,v) \in E$ is a directed edge from u to v, but since the graph is undirected, $(u,v) \in E$ implies that $(v,u) \in E$, too. Signals observed by a node are noisy versions of the sum of all signals transmitted by neighbouring nodes. Due to half-duplex constraints, nodes cannot transmit and receive at the same time. We assume that channel codes exist that allow to reliably communicate. All symbols and all operations are from the finite field $\mathbb{F}_q$, where q can be chosen appropriately. The unit of information is taken as $\log_2 q$ bits, *i.e.*, the base of the logarithm in entropy and mutual information measures is q.

Time is slotted. Let $x_v[t]$ and $y_v[t]$ be the symbols transmitted and reliably received respectively, by node v in time slot t. For $S \subseteq V$, let $x^S[t] = \{x_v[t] | v \in S\}$, with $y^S[t]$ defined similarly. The half-duplex constraints are modelled by extending the input alphabet with a symbol σ denoting that a node is not transmitting. Moreover, we assume that if a node can not receive due to interference or half-duplex constraints, the output y_v is uniformly distributed over $\mathbb{F}_q$. This means that v does not get any information.¶ We restrict our attention to transmission strategies in which the transmission schedule is fixed ahead of time, *i.e.*, strategies for which $P(x_v[t] = \sigma) \in \{0,1\}$.

Symbols received at $v \in V$ depend only on symbols transmitted by neighbouring nodes in the same time slot, *i.e.*,

$$p\left(y_v[t] | x^V[t], x^V[t-1], \ldots\right) = p\left(y_v[t] | x^{N_v}[t]\right), \tag{1}$$

where the conditional probability distribution is constant over time and

$$N_v = \{w \in V | (v,w) \in E\} \cup \{v\}. \tag{2}$$

The exact conditional probability distribution is now specified for each of the modes of operation.

CC: Each node that is not transmitting receives the sum of the symbols transmitted by all its neighbours‖, *i.e.*,

$$p_{\mathsf{CC}}(y_v | x^{N_v}) = \begin{cases} 1, & \text{if } \exists u \in N_v\text{: } x_u \neq \sigma, y_v = \sum_{u \in N_v : x_u \neq \sigma} x_u, x_v = \sigma, \\ \frac{1}{q}, & \text{if } x_v \neq \sigma \text{ or } \forall u \in N_v\text{: } x_u = \sigma, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

¶Another way to model collisions due to interference, would be to extend the output alphabet with an erasure symbol. This, however, creates a covert channel.

‖In general, there are non-ideal effects such as noise and fading, and one can only implement this model approximately. Details can be found in [7,8].

PP: A point-to-point transmission from u to v prevents other transmissions from u, as well as other transmissions to v. For notational convenience, we introduce, for each node $v \in V$, a variable A_v that denotes the neighbour that v is transmitting to. Now,

$$p_{\mathsf{PP}}(y_v|x^{N_v}, a^{N_v}) = \begin{cases} 1, & \text{if } \exists u \in N_v : (y_v = x_u, a_u = v, \forall w \in N_v \setminus \{u\}\text{: } x_w = \sigma), \\ \frac{1}{q}, & \text{if } x_v \neq \sigma \text{ or } \forall u \in N_v\text{: } (x_u = \sigma \text{ or } a_u \neq v) \\ & \quad \text{or } \exists u, w \in N_v\text{: } (u \neq w, x_u \neq \sigma, x_w \neq \sigma), \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

The traffic pattern that we consider is multiple unicast. For a set of K unicast sessions, let S_k and D_k denote the source and destination respectively, of the kth session, and R_k its throughput. Each subset $\Gamma \subseteq V$ of nodes induces a partition of V and hence a directed cut. We will, therefore, often refer to a set of nodes as a cut. For $\Gamma \subseteq V$, let $\bar{\Gamma} = V \setminus \Gamma$ and

$$K_\Gamma = \{k | S_k \in \Gamma, D_k \notin \Gamma\}. \tag{5}$$

Our measure of interest is the *transport capacity* of a network which is defined as the maximum, over all configurations of unicast sessions on a given network and all possible transmission strategies, of $\sum_{k=1}^{K} \text{dist}(S_k, D_k) R_k$, where $\text{dist}(S_k, D_k)$ is the number of hops on the shortest path from S_k to D_k, *i.e.*, the transport capacity is the maximum number of bits×distance that can be transported in the network per unit time. We will derive expressions for the benefit of CC over PP, which we define, for a given network, as the ratio of the transport capacity under CC and the transport capacity under PP.

3 Interference Relations

We capture some of the structure of the topology and the communication models in binary relations $\mathcal{J}_{\mathsf{CC}}$ and $\mathcal{J}_{\mathsf{PP}}$ between edges in $G(V, E)$. If no confusion can arise, or if both relations apply, we will write $\mathcal{J}$ to denote any of these. The relations capture the idea that if there is information being transmitted from u to v and $\left\langle (u, v), (u', v') \right\rangle \in \mathcal{J}$, there can be no information transmitted from u' to v'. This will be made more precise in Lemmas 2 and 3, but first we define the relations. Let $(u, v) \neq (u', v')$ from E.

$$\left\langle (u, v), (u', v') \right\rangle \in \mathcal{J}_{\mathsf{CC}} \text{ iff } v = u' \text{ or } v' = u, \tag{6}$$

$$\left\langle (u, v), (u', v') \right\rangle \in \mathcal{J}_{\mathsf{PP}} \text{ iff } u = u' \text{ or } v \in N_{u'} \text{ or } v' \in N_u. \tag{7}$$

Lemma 1. *$\mathcal{J}$ is symmetric. More precisely, let $(u, v) \neq (u', v')$ from E be given. Then*

$$\left\langle (u, v), (u', v') \right\rangle \in \mathcal{J} \Longleftrightarrow \left\langle (u', v'), (u, v) \right\rangle \in \mathcal{J}. \tag{8}$$

Proof. Immediate. □

The operational meaning of the relations is made precise in the following two lemmas.

Lemma 2. *If $\langle (u, v), (u', v') \rangle \in \mathcal{J}_{\mathsf{CC}}$, then for any joint distribution on X^V satisfying $P(X_w = \sigma) \in \{0, 1\}$ for all $w \in V$ and any subsets $U, W, U', W' \subseteq V$*

$$I(X_u; Y_v | X^U, Y^W) > 0 \Longrightarrow I(X_{u'}; Y_{v'} | X^{U'}, Y^{W'}) = 0. \tag{9}$$

Proof. Assume w.l.o.g. that $u' = v$. Suppose, that both $I(X_u; Y_v | X^U, Y^W) > 0$ and $I(X_{u'}; Y_{v'} | X^{U'}, Y^{W'}) > 0$. This is only possible if both $P(X_u = \sigma) = 0$ and $P(X_{u'} = \sigma) = 0$ (since, by assumption, both these probabilities can only be either zero or one). However, since $u' = v$, it follows from modelling assumption (3), that $Y_{v'}$ is independent of $X_{u'}$, and thus, that $I(X_{u'}; Y_{v'} | X^{U'}) = 0$, which is a contradiction. □

Along similar lines we can show the following result.

Lemma 3. *If $\langle (u, v), (u', v') \rangle \in \mathcal{J}_{\mathsf{PP}}$ then for any joint distribution on X^V and A^V satisfying $P(X_w = \sigma) \in \{0, 1\}$ for all $w \in V$ and any subsets $U, W, U', W' \subseteq V$*

$$I(X_u, A_u; Y_v | X^U, A^U, Y^W) > 0 \Longrightarrow I(X_{u'}, A_{u'}; Y_{v'} | X^{U'}, A^{U'}, Y^{W'}) = 0. \qquad (10)$$

Note, that $G(E, \mathcal{J})$ is very similar to the conflict graph, introduced in [10].

4 Line Network

The results presented in this section are similar to some of the results presented in [5]. The modelling assumptions and proof techniques are different, however. We consider the line network represented by $G(V, E)$, where

$$V = \{0, \ldots, L\}, \quad E = \{(u, v) \subseteq V \times V | \, |u - v| = 1\}. \qquad (11)$$

Theorem 1. *The transport capacity of $G(V, E)$ under CC is L, i.e., for any set of unicast sessions $\{(S_k, D_k)\}$, $\sum_k \operatorname{dist}(S_k, D_k) R_k \leq L$, and there exists a set of unicast sessions together with a coding scheme achieving $\sum_k \operatorname{dist}(S_k, D_k) R_k = L$.*

Proof. (Upper bound:) Let a set of unicast sessions and a network coding strategy over T time slots achieving rate R_k for session $k = 1, \ldots, K$, be given. For $i = 0, \ldots, L-1$, let $\Gamma_i = \{0, \ldots, i\}$ and $\mathcal{S} = \{\Gamma_i, \bar{\Gamma}_i | i = 0, \ldots, L-1\}$. Since a unicast session over d hops crosses d cuts from $\mathcal{S}$,

$$\sum_{S \in \mathcal{S}} \sum_{k \in K_S} R_k = \sum_{k=1}^{K} \operatorname{dist}(S_k, D_k) R_k. \qquad (12)$$

We start developing a cut-set bound following the line of proof found in [11, Theorem 14.10.1], for instance. This gives

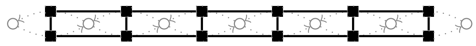
$$\sum_{k \in K_{\Gamma_i}} R_k \leq \frac{1}{T} \sum_{t=1}^{T} I(X^{\Gamma_i}[t]; Y^{\bar{\Gamma}_i}[t] | X^{\bar{\Gamma}_i}[t]). \qquad (13)$$

Summing the LHS and RHS in (13) over all cuts in $\mathcal{S}$ and using (12) give

$$\sum_{k=1}^{K} \operatorname{dist}(S_k, D_k) R_k \leq \frac{1}{T} \sum_{t=1}^{T} \sum_{i=0}^{L-1} [I(X^{\Gamma_i}[t]; Y^{\bar{\Gamma}_i}[t] | X^{\bar{\Gamma}_i}[t]) + I(X^{\bar{\Gamma}_i}[t]; Y^{\Gamma_i}[t] | X^{\Gamma_i}[t])]. \qquad (14)$$

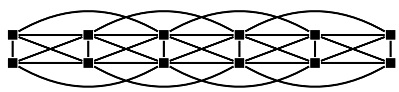
Now, due to the fact that the transmission schedule is fixed ahead of time, $P(X_v[t] = \sigma) \in \{0, 1\}$ for each t and each $v \in V$. We proceed by upper bounding the RHS.

$$\sum_{k=1}^{K} \operatorname{dist}(S_k, D_k) R_k \leq \max_t \sum_{i=1}^{L-1} [I(X^{\Gamma_i}[t]; Y^{\bar{\Gamma}_i}[t] | X^{\bar{\Gamma}_i}[t]) + I(X^{\bar{\Gamma}_i}[t]; Y^{\Gamma_i}[t] | X^{\Gamma_i}[t])]. \qquad (15)$$

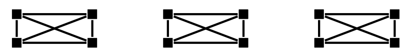


(a) $G(E, \mathcal{J}_{\mathsf{CC}})$ for line network, where E is represented by square shaped nodes and $\mathcal{J}_{\mathsf{CC}}$ by solid lines. Moreover, $G(V, E)$ is depicted by circle shaped nodes and dotted lines.

(b) $G(E, \mathcal{J}')$, where $\mathcal{J}' \subset \mathcal{J}_{\mathsf{CC}}$.



(c) $G(E, \mathcal{J}_{\mathsf{PP}})$ for line network. $G(V, E)$ is now omitted for clarity.



(d) $G(E, \mathcal{J}')$, where $\mathcal{J}' \subset \mathcal{J}_{\mathsf{PP}}$.

Figure 1: Line network. Interference relations under different communication models.

This means, that for any achievable transport capacity, there must exist a joint distribution on X_v, with $P(X_v = \sigma) \in \{0, 1\}$ for each $v \in V$, satisfying

$$\sum_{k=1}^{K} \mathrm{dist}(S_k, D_k) R_k \leq \sum_{i=0}^{L-1} [I(X^{\Gamma_i}; Y^{\bar{\Gamma}_i} | X^{\bar{\Gamma}_i}) + I(X^{\bar{\Gamma}_i}; Y^{\Gamma_i} | X^{\Gamma_i})] \tag{16}$$

$$\leq \sum_{i=0}^{L-1} [I(X_i; Y_{i+1} | X^{\bar{\Gamma}_i}) + I(X_{i+1}; Y_i | X^{\Gamma_i})], \tag{17}$$

where the second inequality follows after decomposing the mutual information terms and using (3).**

We now argue that for all probability distributions of this kind, the right hand side of (17) is upper bounded by L. Since each term individually can be at most one, it is sufficient to show that at most L terms in (17) can be made positive. By Lemma 2, this number is exactly the size of the maximum independent set in the graph $G(E, \mathcal{J}_{\mathsf{CC}})$, which is depicted in Figure 1(a). Since the size of the maximum independent set can not decrease in size by removing some of the links of $G(E, \mathcal{J}_{\mathsf{CC}})$, we consider the graph given in 1(b). Since, this graph consists of disjoint components which are cliques, the maximum independent set is upper bound by the number of components, which is L.

(Lower bound:) We use two unicast sessions, with $S_1 = 0$, $R_1 = L$, $S_2 = L$, $R_2 = 0$. Using techniques demonstrated in [2,5] we can achieve throughput 1/2 for both sessions (Details are omitted due to space constraints). The distance between each source and receiver is L, giving $\sum_{k=1}^{2} |S_k - D_k| R_k = L$. □

Theorem 2. *The transport capacity under* PP *of* $G(V, E)$ *is* $\left\lceil \frac{1}{2} L \right\rceil$*, i.e., for any set of unicast sessions* $\{(S_k, D_k)\}$, $\sum_{k=1}^{K} \mathrm{dist}(S_k, D_k) R_k \leq \left\lceil \frac{1}{2} L \right\rceil$*, and there exists a set of unicast sessions together with a coding scheme achieving* $\sum_k \mathrm{dist}(S_k, D_k) R_k = \left\lceil \frac{1}{2} L \right\rceil$.

**A more common form of the cut-set bound is to introduce a time-sharing variable and perform an averaging argument instead of taking the maximum over t on the RHS. In general, however, the averaged distribution does not satisfy the condition that $P(X_v = \sigma) \in \{0, 1\}$ for all $v \in V$.

Proof. For the upper bound we start from (17) and consider $G(E, \mathcal{J}_{\mathsf{PP}})$ as given in Figure 1(c). Again, by removing links we get 1(d), which consists of disjoint components which are cliques. By writing L as $L = 2\alpha + \beta$, $\alpha, \beta \in \mathbb{N}$, $0 \leq \beta \leq 1$, we see that the maximum independent set is of size at most $\alpha + \beta$, where $\alpha = \lfloor L/2 \rfloor$ and $\beta = L - 2\lfloor L/2 \rfloor$. The lower bound is omitted due to space constraints. □

Theorems 1 and 2 give the following result.

Corollary 1. *The benefit in transport capacity of* CC *over* PP *on the line network is* $L/\lceil L/2 \rceil$*,i.e., it is* 2 *if* L *is even and* $2 - 2/(L+1)$ *if* L *is odd.*

5 Hexagonal Lattice

We consider $G(V, E)$, were V is a subset of size $(L+1) \times (M+1)$ of the hexagonal lattice, with edges between nearest neighbours. We index nodes with a tuple $(u_1, u_2) \in \mathbb{N}^2$. The location in $\mathbb{R}^2$ of (u_1, u_2) is $G_\Lambda \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T$, with $G_\Lambda = \begin{bmatrix} 1 & 0 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$. Now,

$$V = \{(u_1, u_2) | 0 \leq u_1 \leq L, 0 \leq u_2 \leq M\}, E = \{((u_1, u_2), (v_1, v_2)) | \left\| G_\Lambda \begin{bmatrix} u_1 - v_1 \\ u_2 - v_2 \end{bmatrix} \right\| = 1\}.$$

Note, that the number of (directed) edges in $G(V, E)$ is $6LM + 2(L + M)$.

Theorem 3. *The transport capacity of* $G(V, E)$ *under* CC *is upper bounded by* $2(LM + L + M)$*, i.e., for any set of unicast sessions* $\{(S_k, D_k)\}$

$$\sum_k \operatorname{dist}(S_k, D_k) R_k \leq 2(LM + L + M). \tag{18}$$

Moreover, it is lower bounded by $LM - o(LM)$*, i.e., there exists a set of unicast sessions together with a coding scheme achieving* $\sum_k \operatorname{dist}(S_k, D_k) R_k = LM - o(LM)$*.*

Proof. (Sketch for upper bound.) Consider the cuts

$$\Gamma_i^1 = \{(u_1, u_2) \in V | u_1 \leq i\}, \quad i = 0, \ldots, L - 1, \tag{19}$$

$$\Gamma_i^2 = \{(u_1, u_2) \in V | u_2 \leq i\}, \quad i = 0, \ldots, M - 1, \tag{20}$$

$$\Gamma_i^3 = \{(u_1, u_2) \in V | u_1 + u_2 \leq i\}, \quad i = 0, \ldots, L + M - 1. \tag{21}$$

Let

$$\mathcal{S} = \{\Gamma_i^1, \bar{\Gamma}_i^1\}_{i=0}^{L-1} \cup \{\Gamma_i^2, \bar{\Gamma}_i^2\}_{i=0}^{M-1} \cup \{\Gamma_i^3, \bar{\Gamma}_i^3\}_{i=0}^{L+M-1}. \tag{22}$$

Figures 2 and 3 depict $G(V, E)$ and the lines inducing the cuts in $\mathcal{S}$, respectively.

Since on the shortest path between two nodes, the number of cuts crossed on each hop is 2 and no cut is crossed more than once,

$$\sum_{S \in \mathcal{S}} \sum_{k \in K_S} R_k = 2 \sum_{k=1}^{K} \operatorname{dist}(S_k, D_k) R_k. \tag{23}$$

By developing a cut-set bound of the same form as given in the proof of Theorem 1, one can obtain an inequality similar to (17), with (23) on the LHS and, in this case, two mutual information terms for each edge in E on the RHS. Therefore, by Lemma 2, the RHS is upper bounded by twice the size of the maximum independent set in $G(E, \mathcal{J}_{\mathsf{CC}})$. Figure 4 depicts a set of 3 edges forming a clique in $G(E, \mathcal{J}_{\mathsf{CC}})$. Now, most of E can be covered by non-overlapping sets of this type. This will require $2LM$ sets, leaving $2(L + M)$ edges uncovered, hence the size of an independent set is at most $2(LM + L + M)$. The proof of the lower bound is omitted due to space constraints. □

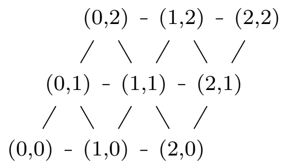


Figure 2: $G(V,E)$ for $L = M = 2$.

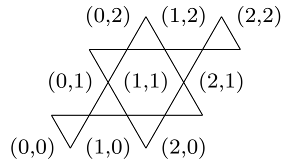


Figure 3: Lines inducing the partitions $\mathcal{S}$.

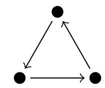


Figure 4: Subgraph of $G(V,E)$, edges forming a clique in $G(E, \mathcal{J}_{\mathsf{CC}})$.

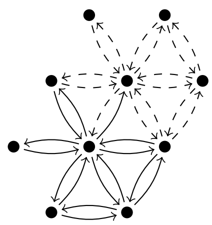


Figure 5: Subgraph of $G(V,E)$. The solid edges form a clique in $G(E, \mathcal{J}_{\mathsf{PP}})$. Also, the set of dashed edges form a clique in $G(E, \mathcal{J}_{\mathsf{PP}})$.

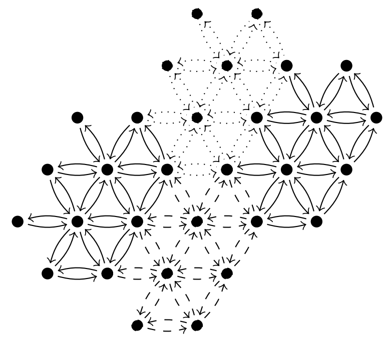


Figure 6: Tiling the set of edges depicted in Figure 5 in such a way that all edges in $G(V,E)$ are covered exactly once.

Theorem 4. *The transport capacity of $G(V,E)$ under* PP *is at most $\frac{2}{5}LM + o(LM)$. More precisely, for any set of unicast sessions $\{(S_k, D_k)\}$*

$$\sum_k \mathrm{dist}(S_k, D_k) R_k \leq \frac{2}{5} LM + o(LM). \tag{24}$$

Moreover, it is at least $\frac{1}{3}LM - o(LM)$, i.e., there exists a set of unicast sessions together with a coding scheme achieving $\sum_k \mathrm{dist}(S_k, D_k) R_k = \frac{1}{3}LM - o(LM)$.

Proof. (Sketch for upper bound.) Consider the set of edges depicted in Figure 5. The set is partitioned in two, such that the edges in each partition form a clique in $G(E, \mathcal{J}_{\mathsf{PP}})$. This means that the size of the maximum independent set of this subset in $G(E, \mathcal{J}_{\mathsf{PP}})$ is 2. Also, the set of edges from Figure 5 can be tiled around in such a way that all edges in $G(V,E)$ are covered exactly once. This is depicted in Figure 6. The number of edges in Figure 5 is 30. Therefore, the size of the maximum independent set of the whole of $G(E, \mathcal{J}_{\mathsf{PP}})$ is approximately $6LM/15 = 2LM/5$, where the approximation comes from boundary effects. The proof of the lower bound is omitted due to space constraints. □

Theorems 3 and 4 give the following bounds on the benefit of CC over PP.

Corollary 2. *The benefit in transport capacity of* CC *over* PP *on the hexagonal lattice is at least $2.5 - o(LM)$ and at most $6 + o(LM)$.*

6 Discussion

We have considered the transport capacity of the line network and the hexagonal lattice under point-to-point and computation coding strategies. Our main result is that in networks with nodes positioned at the hexagonal lattice, the benefit of multi-access computation coding over point-to-point communication is at least 2.5. This improves upon the previously known lower bound of 2 on the benefit of multi-access computation coding. In future work we intend to analyze other coding strategies, *e.g.*, exploiting broadcast, but not multi-access, as well as other networks.

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