On the Energy Savings of Network Coding in Wireless Networks

Jasper Goseling* Jos H. Weber
IRCTR/CWPC, WMC Group
Delft University of Technology
The Netherlands
`{j.goseling, j.h.weber}tudelft.nl`

Abstract

The energy consumption of a wireless device is modeled by including not only the energy emitted while transmitting, but also energy consumed by supporting circuitry. In particular also receiver energy consumption is taken into account. It is shown that under this model, compared to traditional routing, the energy reduction offered by network coding is significantly different from results reported in the literature based on an energy consumption model that includes the energy emitted while transmitting only. Moreover, it is illustrated that energy can be saved by increasing the transmission power. Whereas this causes individual transmissions to consume more energy, overall energy consumption can be reduced since more coding opportunities arise.

1 Introduction

The battery lifetime of a device is a critical factor in many applications of wireless networks. Therefore, it is of utmost importance to keep the energy consumption in wireless networks as low as possible. An important contribution to the energy consumption is formed by the energy that is emitted while transmitting. Therefore, optimizing the transmit power in multi-hop networks has been the topic of many studies, for instance [1–3]. By considering only the transmitted energy, it is concluded that it is optimal to keep the transmit power as low as possible, while ensuring connectivity. Hence, one should take many short hops instead of less hops over a larger distance.

There are applications in which the transmit power is so large that the energy consumption is dominated by the transmit energy. However, there are also many applications in which the transmit power is relatively low and other contributions to the energy consumption can not be neglected, for instance, in wireless sensor networks. The other contributions consist of, for instance, energy consumed by supporting circuitry while transmitting, as well as energy consumed while receiving. Examples of exact values of the energy consumed while transmitting and receiving at various transmit power levels can be found in the literature [4] as well as in manufacturer specifications of devices, *e.g.*, [5]. These values demonstrate that: 1) energy consumed while receiving is not always negligible, and 2) transmitter energy consumption is not always dominated by the transmitted energy. In [6] this observation has been used to demonstrate that it is not always optimal to take the shortest possible hops. Indeed, if, independent of the hop distance, a large amount of energy is consumed in each hop, it might be optimal to take few long hops. In the current paper we will consider the consequence of the above observations on the energy consumption in wireless networks in which network coding (*cf.* [7]) is employed. We will reconsider some of the examples presented in the literature for which it has been demonstrated that significant reduction in energy consumption

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[*] Also with Department of Applied Mathematics, University of Twente, The Netherlands.

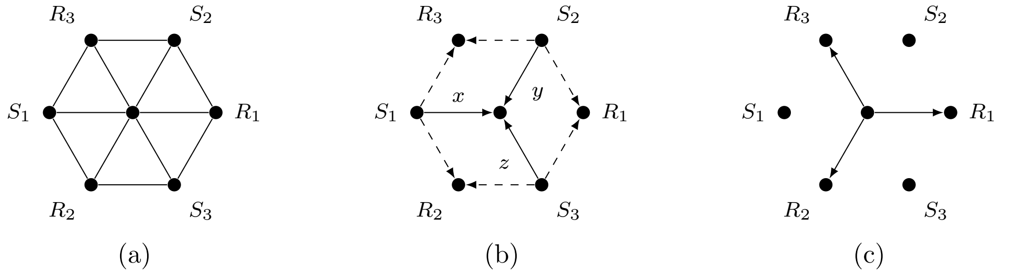


Figure 1: Network coding solution to three session multiple unicast configuration. Configuration and connectivity in (a). Transmissions from sources in (b). Note that each transmission is received by three nodes. The center nodes transmits $x + y + z$, see (c).

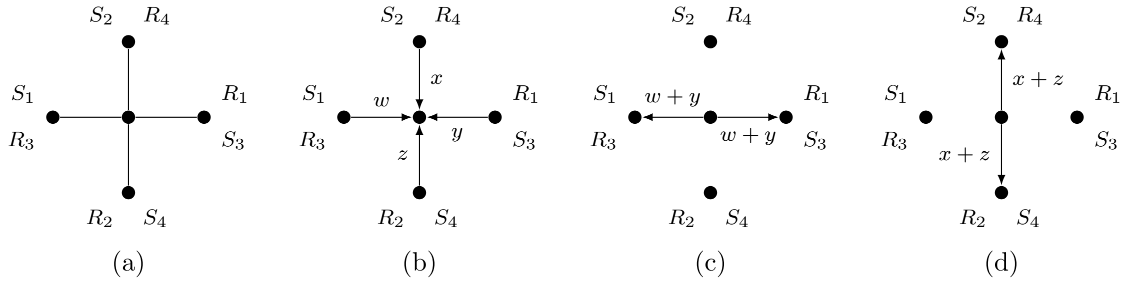


Figure 2: Network coding solution to the four session multiple unicast configuration depicted in (a). The solution depicted in (b)–(d) performs coding for the horizontal and vertical sessions independently and uses a total of 8 transmissions.

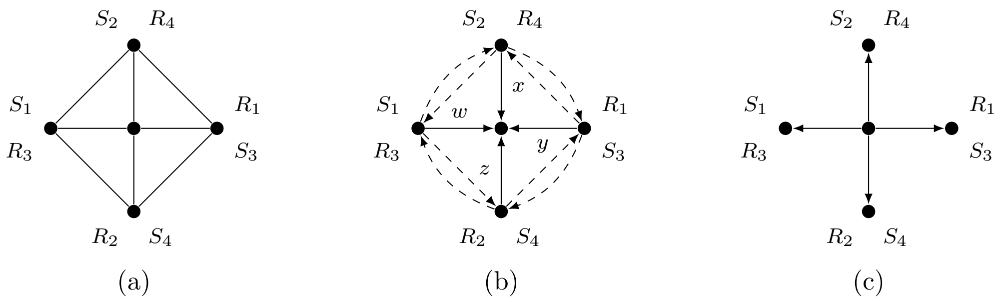


Figure 3: The four session configuration from Figure 2 with increased transmission range leading to connectivity as in (a). The network coding solution depicted in (b) and (c) uses 5 transmissions.

can be achieved by using network coding. For these examples we will analyze the use of network coding under a more detailed energy consumption model. In addition we will consider an increased transmission range for reducing energy consumption.

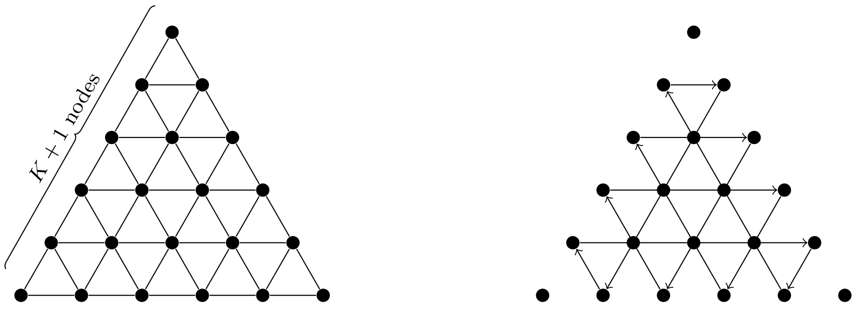
The reduction in energy consumption that can be obtained by using network coding is based on the fact that the broadcast effect of the wireless medium allows multiple nodes to receive the same transmission. By properly encoding messages, one transmission can be useful for several receivers. We illustrate this by means of the example from Figure 1. The figure depicts a three session unicast problem for which a solution without network coding would require six transmissions in order to transfer one symbol for each session. The network coding solution uses only four transmissions. The last transmission $x+y+z$ by the center node, depicted in Figure 1c, is useful for all sinks. Note that, besides this transmission, node R_1 has previously received symbols y and z. Therefore, R_1 can recover x as $x = (x+y+z) - y - z$. The other sinks can decode in similar fashion. The first contribution of this paper deals with the observation that the network coding solution requires each sink to receive three symbols, whereas without network coding only one symbol needs to be received. Hence, there is a tension between the energy saved by reducing the numbers of transmissions and the additional energy spend by the sinks in receiving more symbols. In the remainder of this paper we will explore this tradeoff in more detail.

The second observation made in the current paper is that by increasing the transmission power, the transmission range is increased and more nodes are able to receive each message, hence potentially creating coding opportunities and enabling more efficient operation. Again, we illustrate by means of an example. In Figure 2 we have depicted a configuration with four multicast sessions and a coding solution in which the center nodes transmits twice. Note that this coding solution is using the well-known piggy-backing strategy [8] for the horizontal and vertical sessions independently. In Figure 3 we have depicted the topology resulting from an increased transmission range and a corresponding network coding solution in which the center node is transmitting only once. Hence the number of transmissions is reduced by increasing the transmission range. Obviously, there is a tension between the increased efficiency and the additional energy spend by increasing the transmit power. Moreover, receiver energy consumption is increased since more transmissions need to be overheard.

This paper is organized as follows. The model will be defined more precisely in Section 2. In Section 3 we will analyze the performance of some network coding solutions for specific multiple unicast configurations that have been proposed in the literature. It is known that, compared to traditional routing, these coding solutions greatly reduce the energy emitted by the antennas. We will analyze the overall energy consumption of these coding solutions. In Section 4 we will analyze the impact of the transmission range on overall energy consumption. It will be shown that there are scenarios for which increasing the transmission power reduces overall energy consumption. Finally, the results of the paper will be discussed in Section 5.

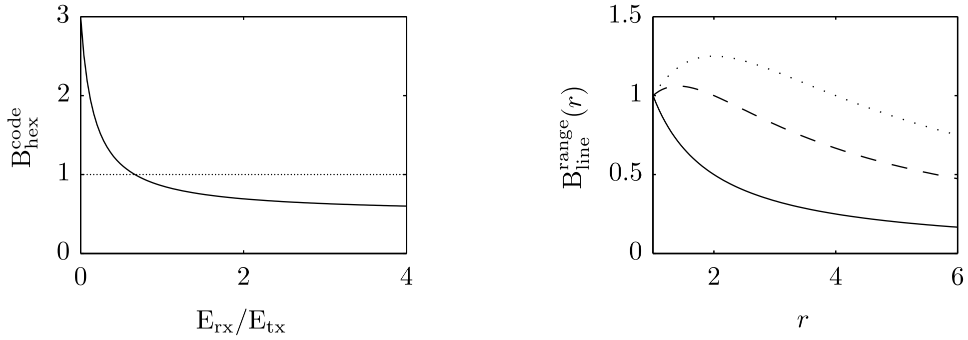
2 Model

All configurations considered in this paper are multiple unicast, *i.e.*, there are several sessions in which a single source is communicating to a single destination. Energy consumption will be measured by the average energy neccesary for delivering one symbol for each of the sessions, where the average is over the nodes in the network, the unicast sessions and time. Energy consumption of a device is separated in two quantities: $\mathrm{E}_{\mathrm{tx}}(r)$ and E_{rx} denoting the energy required to transmit a symbol at transmission range r and the energy required to receive a symbol respectively. Since we are interested in energy consumption only, we can schedule all transmissions sequentially. Hence, we can assume that there is no interference. W.l.o.g. we assume that all symbols are bits, *i.e.*, elements from $\mathbb{F}_2$, the finite field with two elements. We assume that signals are



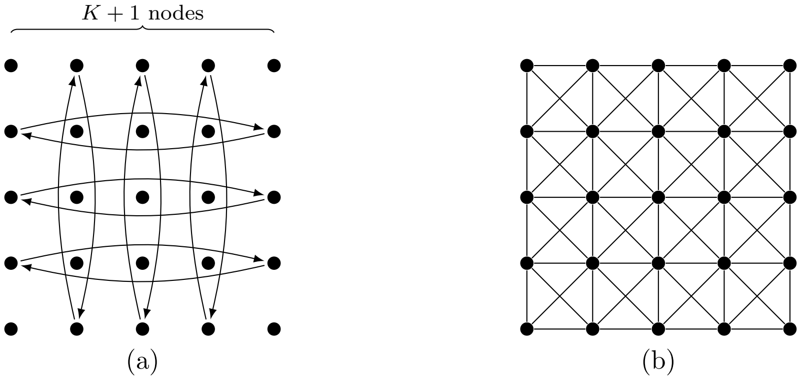
(a) (b)

Figure 4: Multiple unicast configuration with nodes placed at the hexagonal lattice and connectivity as depicted in (a). Unicast sessions are placed according to (b).



(a) (b)

Figure 5: In (a): The energy benefit of network coding for the configuration from Figure 4 as a function of $E_{\text{rx}}/E_{\text{tx}}$. In (b): The benefit of increasing the transmission range in the line network for $\alpha = 2$ and $\beta = 0, 2, 4$ in solid, dashed and dotted lines respectively.



(a) (b)

Figure 6: Nodes located at integer coordinates. Unicast sessions are placed according to (a). Connectivity based on $r = \sqrt{2}$ is depicted in (b).

attenuated exponentially over distance with path loss exponent α. Moreover, we assume a fixed transmission rate. Hence, in order to have correct reception of a message, we require the signal to noise ratio to be above a certain threshold, *i.e.*, to have transmit power proportional to r^α, where r is the distance over which transmission takes place. We assume that each node within transmission range r is capable of receiving the transmitted symbol without error. Besides the energy that is emitted while transmitting, there is an additional part independent of the transmission range. This leads to the following relation between the transmission range and the energy consumption per transmitted symbol at the transmitter

$$\mathrm{E}_{\mathrm{tx}}(r) = \kappa_2(\kappa_1 + r^\alpha), \tag{1}$$

where κ_1 and κ_2 are constants. The values of the constants depend on the hardware and the communication protocols that are being used. In this paper we will not deal with an interpretation of or exact values for these parameters. Since we will compare the energy consumption of various routing and coding schemes their values will only enter the analysis by means of the relation between the different parameters. We comment here on the values that can be expected in practical devices. Depending on the operating regime, the value of κ_1 can range from being negligible compared to r^α, to being of the same order of magnitude or even larger than r^α. The specifications of a device in which κ_1 is larger than r^α are given in [5]. In addition similar measurements are presented in [4] for other devices. Also the value of E_{rx} can range from being negligible compared to r^α, to being larger than r^α, again see, *e.g.*, [4, 5].

3 To code or not to code

In this section we will consider the ratio of the minimum energy required by routing and network coding solutions for two different families (indexed by parameter $K \in \mathbb{N}$ denoting the network size) of configurations, the line network and the hexagonal lattice. We call the limit of this ratio as $K \to \infty$ the energy benefit of network coding and denote it by $\mathrm{B}^{\mathrm{code}}_{\mathrm{line}}$ and $\mathrm{B}^{\mathrm{code}}_{\mathrm{hex}}$ for the line network and the hexagonal lattice respectively.

We start this section with a discussion on information exchange in the network of K uniformly spaced nodes on a line with interdistance one. There are two oppositely directed unicast sessions with sources and destinations at the boundaries of the network. We assume that in both the network coding and routing case $r = 1$. For notational convenience, the dependence of E_{tx} on r will be left implicit. Note that a routing solution requires all nodes except for the two boundary nodes to transmit and receive twice. In addition, it is known that a network coding solution exists that uses one transmission at each of those nodes using a piggy-backing strategy [8], again each node needs to receive twice. Since we consider $K \to \infty$ we can ignore the nodes at the boundary. It follows that the energy benefit, $\mathrm{B}^{\mathrm{code}}_{\mathrm{line}}$ is

$$\mathrm{B}^{\mathrm{code}}_{\mathrm{line}} = \frac{2\mathrm{E}_{\mathrm{tx}} + 2\mathrm{E}_{\mathrm{rx}}}{\mathrm{E}_{\mathrm{tx}} + 2\mathrm{E}_{\mathrm{rx}}} = \frac{2 + 2\frac{\mathrm{E}_{\mathrm{rx}}}{\mathrm{E}_{\mathrm{tx}}}}{1 + 2\frac{\mathrm{E}_{\mathrm{rx}}}{\mathrm{E}_{\mathrm{tx}}}}. \tag{2}$$

If $\mathrm{E}_{\mathrm{rx}} = 0$ we see that $\mathrm{B}^{\mathrm{code}}_{\mathrm{line}} = 2$. However, if $\mathrm{E}_{\mathrm{rx}} = \mathrm{E}_{\mathrm{tx}}$, for instance, this reduces to $\mathrm{B}^{\mathrm{code}}_{\mathrm{line}} = \frac{4}{3}$, which differs significantly from the benefit of 2 which is usually presented in the literature. Finally, note that the ratio is always larger than one, *i.e.*, the network coding solution will always perform better than the routing solution. In the remainder of this section we will see that there are also configurations in which, for certain ranges of parameter values, known coding solutions perform worse than routing solutions.

The second example that we consider in this section is the extension of Figure 1 to a larger network. Configuration K has $(K + 2)(K + 1)/2$ nodes positioned at the hexagonal lattice and $3(K-1)$ unicast sessions as depicted in Figure 4. This configuration was previously considered in [9]. Note that the number of nodes at the border of the network is $\Theta(K)$, while the number of nodes in the interior is $\Theta(K^2)$. Hence, since we consider $K \to \infty$, if the energy consumed by a node at the border is independent of K, we can consider only the energy consumption in the interior of the network and neglect the borders. First note that the optimal routing solution is transmitting data along the shortest paths between sources and destinations. Hence, a routing solution requires each interior node to transmit and receive three times. Next, in order to bound the energy consumption if network coding is allowed we make use of results from [9]. In [9] a network code was presented in which each interior node is transmitting once in order to deliver a symbol for each session. Moreover each node at the border is transmitting twice, hence this contribution to the energy consumption can be neglected if $K \to \infty$. In [9] we did not consider receiver energy consumption. Inspection of the proposed code shows that each interior node needs to receive six times. Note, finally that the energy consumption of this particular network code forms an upper bound to the minimum energy required by any network code. Hence the benefit can be lower bounded as

$$\mathrm{B}_{\mathrm{hex}}^{\mathrm{code}} \geq \frac{3\mathrm{E}_{\mathrm{tx}} + 3\mathrm{E}_{\mathrm{rx}}}{\mathrm{E}_{\mathrm{tx}} + 6\mathrm{E}_{\mathrm{rx}}} = 3\frac{1 + \frac{\mathrm{E}_{\mathrm{rx}}}{\mathrm{E}_{\mathrm{tx}}}}{1 + 6\frac{\mathrm{E}_{\mathrm{rx}}}{\mathrm{E}_{\mathrm{tx}}}}. \tag{3}$$

This recovers the result from [9] that if $\mathrm{E}_{\mathrm{rx}} = 0$ then $\mathrm{B}_{\mathrm{hex}}^{\mathrm{code}} \geq 3$. Moreover we can conclude that we have a benefit larger than one if $\mathrm{E}_{\mathrm{tx}} > 3\mathrm{E}_{\mathrm{rx}}/2$. However, for $\mathrm{E}_{\mathrm{tx}} \leq 3\mathrm{E}_{\mathrm{rx}}/2$ no code is known that achieves better than routing. We have plotted this bound on $\mathrm{B}_{\mathrm{hex}}^{\mathrm{code}}$ as a function of $\mathrm{E}_{\mathrm{rx}}/\mathrm{E}_{\mathrm{tx}}$ in Figure 5a.

The main message of this section is that taking receiving energy consumption into account can significantly influence results on the energy benefit of network coding.

4 Transmission range

In this section we consider the influence of the transmission power on the overall energy consumption. We consider two families of configurations, the line network that was also used in the previous section and a rectangular lattice to be defined more precisely later in this section.Let $\mathrm{B}_{\mathrm{line}}^{\mathrm{range}}(r)$ and $\mathrm{B}_{\mathrm{rect}}^{\mathrm{range}}(r)$ denote the benefit of increasing from transmission range 1 to r for the line and rectangular lattice network respectively. More precisely, the benefit is defined as the ratio of the minimum energy consumption at ranges 1 and r in the limit $K \to \infty$. We start with the line network for which we have seen in the previous section that independent of the value of $\mathrm{E}_{\mathrm{rx}}/\mathrm{E}_{\mathrm{tx}}$ it is beneficial to use network coding. Therefore, we will compare the energy consumption of the network coding solution given in the previous section at various transmission powers, *i.e.*, at ranges 1 and r. Observe that at transmission range r only every $\lfloor r \rfloor th$ needs to participate in a coding scheme. The other nodes can remain inactive. Hence, it is straightforward to derive

$$\mathrm{B}_{\mathrm{line}}^{\mathrm{range}}(r) = \frac{\lfloor r \rfloor \left(\mathrm{E}_{\mathrm{tx}}(1) + 2\mathrm{E}_{\mathrm{rx}}\right)}{\mathrm{E}_{\mathrm{tx}}(r) + 2\mathrm{E}_{\mathrm{rx}}} = \frac{\lfloor r \rfloor (1+\beta)}{r^{\alpha} + \beta}, \tag{4}$$

where, for notational convenience, we have introduced

$$\beta = \kappa_1 + 2\mathrm{E}_{\mathrm{rx}}/\kappa_2. \tag{5}$$

In order to characterize the behavior of $\mathrm{B}_{\mathrm{line}}^{\mathrm{range}}(r)$ we analyze

$$\tilde{\mathrm{B}}_{\mathrm{line}}^{\mathrm{range}}(r) = \frac{r(1+\beta)}{r^{\alpha} + \beta}, \tag{6}$$

which is an upper bound to $\mathrm{B}_{\text{line}}^{\text{range}}(r)$. The maximum of $\tilde{\mathrm{B}}_{\text{line}}^{\text{range}}(r)$ is attained at

$$r = \sqrt[\alpha]{\frac{\beta}{\alpha - 1}}. \tag{7}$$

Finally, we determine the values of β for which $\tilde{\mathrm{B}}_{\text{line}}^{\text{range}}(r) \leq 1$ for all $r \geq 1$, *i.e.*, for which it is not beneficial to increase the transmission range. From $\tilde{\mathrm{B}}_{\text{line}}^{\text{range}}(1) = 1$, the value of the derivative and continuity, it follows that

$$\beta < \alpha - 1 \Rightarrow \tilde{\mathrm{B}}_{\text{line}}^{\text{range}}(r) \leq 1, \quad r \geq 1. \tag{8}$$

We have plotted the values of $\tilde{\mathrm{B}}_{\text{line}}^{\text{range}}(r)$ as function of r for $\alpha = 2$ and various values β in Figure 5b. In Figure 5b we have used $\alpha = 2$ in which case the above expression for the maximizing transmission range, Equation (7), reduces to $r = \sqrt{\beta}$.

Next, we consider the energy consumption in the family (indexed by parameter $K \in \mathbb{N}_{>1}$) of multiple unicast configurations depicted in Figure 6. Configuration K has $(K+1)^2$ nodes located at the integer coordinates in $[0, K]^2$. Of these nodes there are $\Theta(K^2)$ nodes in the interior and $\Theta(K)$ nodes at the border of the network. There are $4(K-1)$ unicast sessions. First, note that for each pair of oppositely directed sessions it is possible to use the line network information exchange network coding scheme that was discussed in the previous section. From this discussion it follows that network coding always reduce energy consumption. Therefore, we will compare network coding solutions at various transmission ranges. In constrast to the first part of this section we will compare $r = 1$ and $r = \sqrt{2}$ only. At $r = 1$ we use the information exchange scheme from [8] for pairs of oppositely directed sessions. Hence each interior node needs to transmit twice and receive four times.

For $r = \sqrt{2}$ an efficient network code was presented in [9]. Note that at $r = \sqrt{2}$ each node in the interior of the network has 8 neighbours. In order to deliver one symbol for each of the sessions, the code from [9] requires each interior node to receive from all 8 neighbours. In addition one transmission is required. The nodes at the border transmit twice and receive at most eight times. Hence, the contributions from the border nodes can be neglected. It follows that

$$\mathrm{B}_{\text{rect}}^{\text{range}}(\sqrt{2}) \geq \frac{2\mathrm{E}_{\text{tx}}(1) + 4\mathrm{E}_{\text{rx}}}{\mathrm{E}_{\text{tx}}(\sqrt{2}) + 8\mathrm{E}_{\text{rx}}} = \frac{2\kappa_1 + 2 + 4\mathrm{E}_{\text{rx}}/\kappa_2}{\kappa_1 + 2^{\alpha/2} + 8\mathrm{E}_{\text{rx}}/\kappa_2}. \tag{9}$$

It follows that the benefit is larger than one if

$$\mathrm{E}_{\text{rx}} < \frac{\kappa_2 \left(\kappa_1 + 2 - 2^{\alpha/2}\right)}{4}. \tag{10}$$

This means that in the configuration depicted in Figure 6 it is beneficial to increase the transmission range from $r = 1$ to $r = \sqrt{2}$ if the above condition is satisfied. Note, that increasing from $r = 1$ to $r = \sqrt{2}$ does not affect the number of hops on the shortest path between any source and destination. Hence, the reduced energy consumption comes from more efficient coding that is possible at $r = \sqrt{2}$. In contrast, in the line network, as discussed in the first part of this section, lower energy consumption arose from a reduction in the number of hops.

5 Discussion

We have analyzed the energy consumption of several network coding solutions to wireless multiple unicast problems. The model that has been used includes receiver energy consumption as well as energy consumption of supporting circuitry while transmitting. It has been demonstrated that under this model the benefit of using these coding solutions can be significantly different from results reported in the literature based on models that include only the energy emitted while transmitting. Moreover, it has been shown that by increasing the transmission power it is possible to reduce the overall energy consumption in the network since more coding opportunities are created.

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