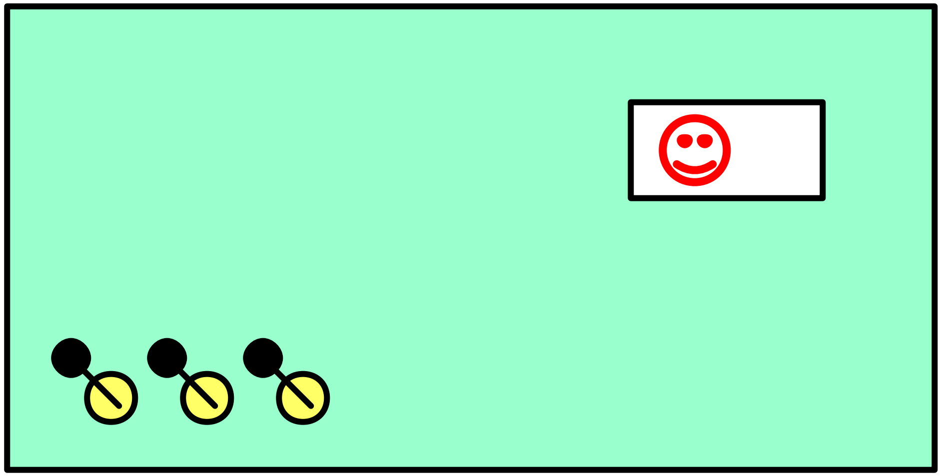
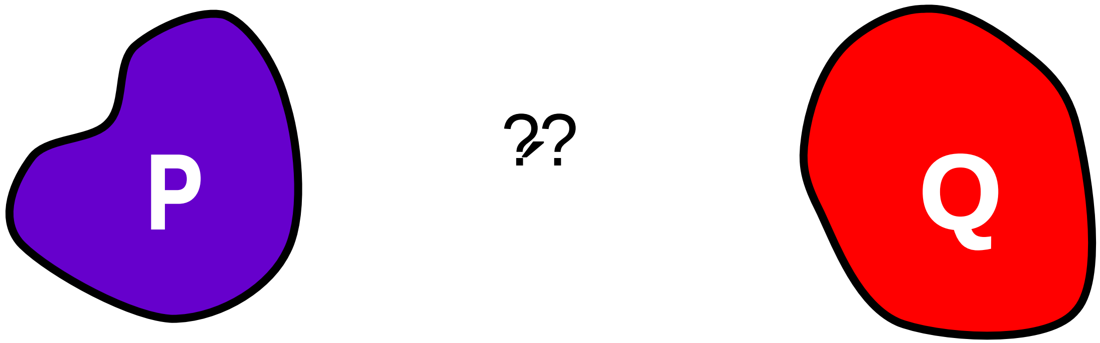
A testing scenario for probabilistic automata



Marielle Stoelinga
UC Santa Cruz
Frits Vaandrager
University of Nijmegen

Characterization of process equivalences

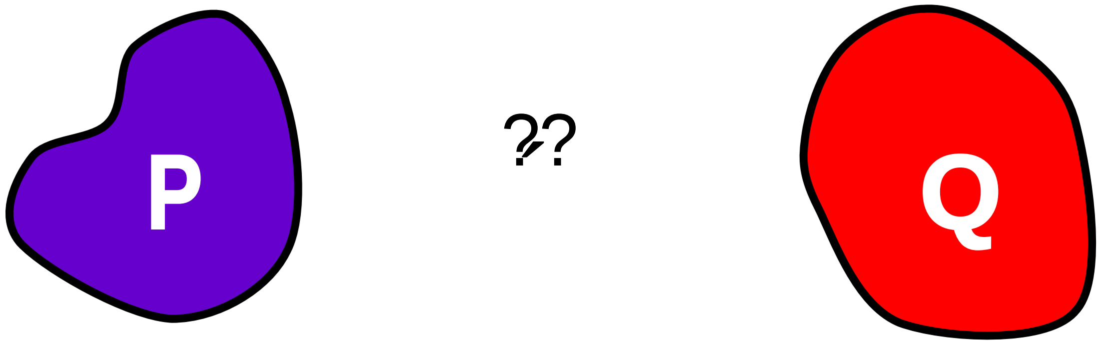


- trace (language) equivalence
- bisimulation equivalence
- ready trace equivalence

  ....

- **comparative concurrency semantics [DH..,Mil80,vGl01]**
  - compare various equivalences
  - justify equiv via testing scenarios / button pushing experiments

Characterization of process equivalences



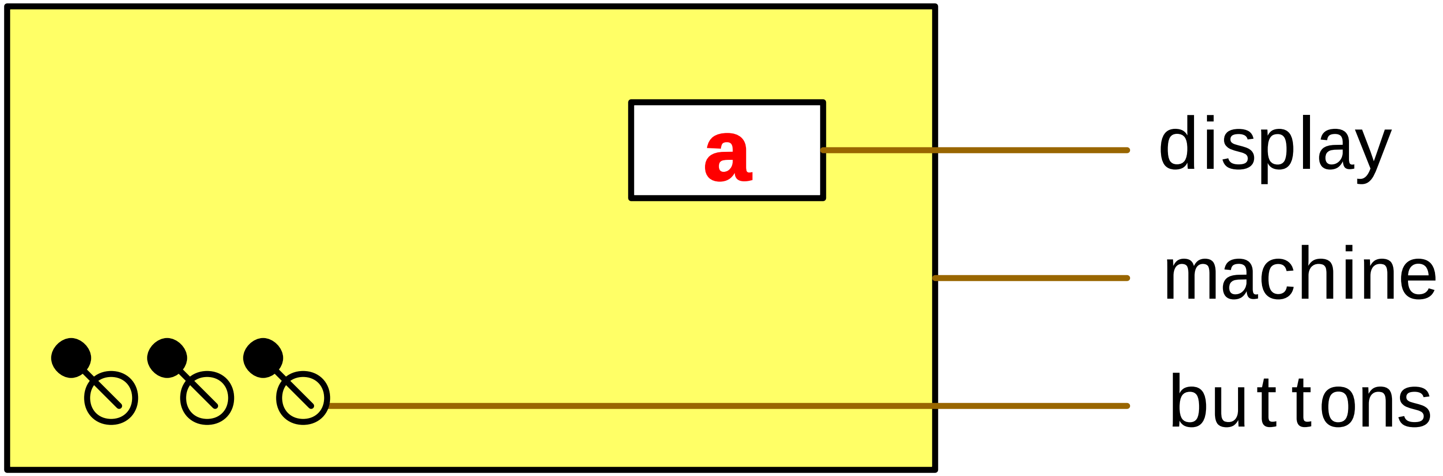
- testing scenarios:
  - define intuitive notion of **observation,** fundamental
  - processes that cannot be distinguished by observation are deemed to be equivalent
  - justify process equivalence ´

    P ´ Q iff Obs(P) = Obs(Q)
  - ´ does not distinguish too much/too little

Testing scenario

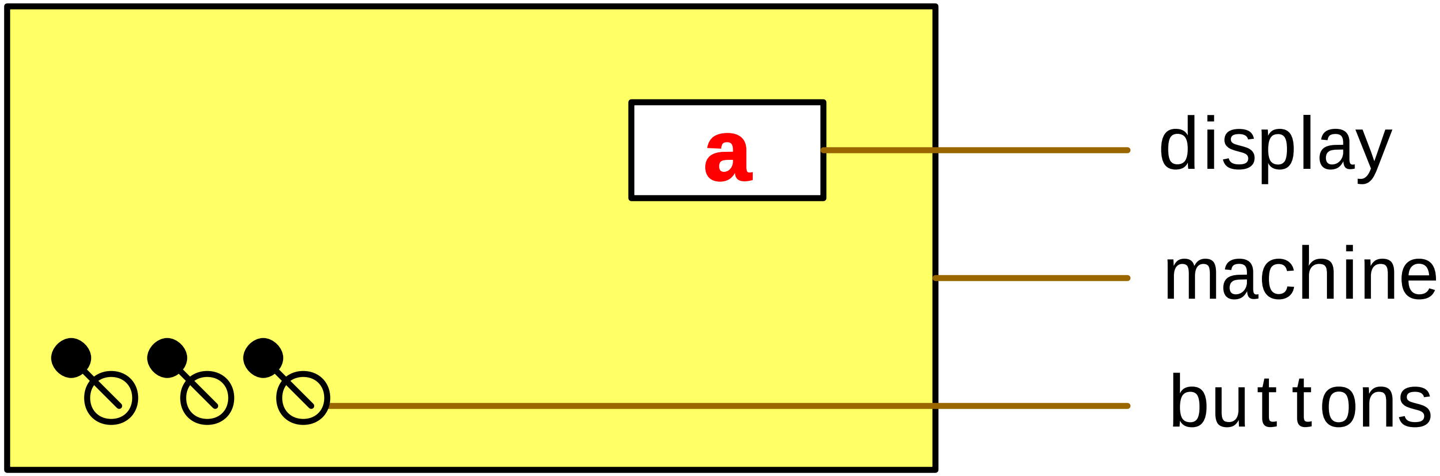
- testing scenario's in non-probabilistic case
  - trace equivalence
  - bisimulation
  - ...
- we define
  - observations of a PA
  - observe probabilities through statistical methods (hypothesis testing)
- main result
  - Obs(P) = Obs(Q) iff trd(P) = trd(Q), P, Q fin branching
    - trd(P) extension of traces for PAs. [Segala]
    - justifies trace distr equiv in terms of observations

Model for testing scenarios



- machine M
  - a black box
  - inside: process described by LTS P
- display
  - showing current action
- buttons
  - for user interaction

Model for testing scenarios

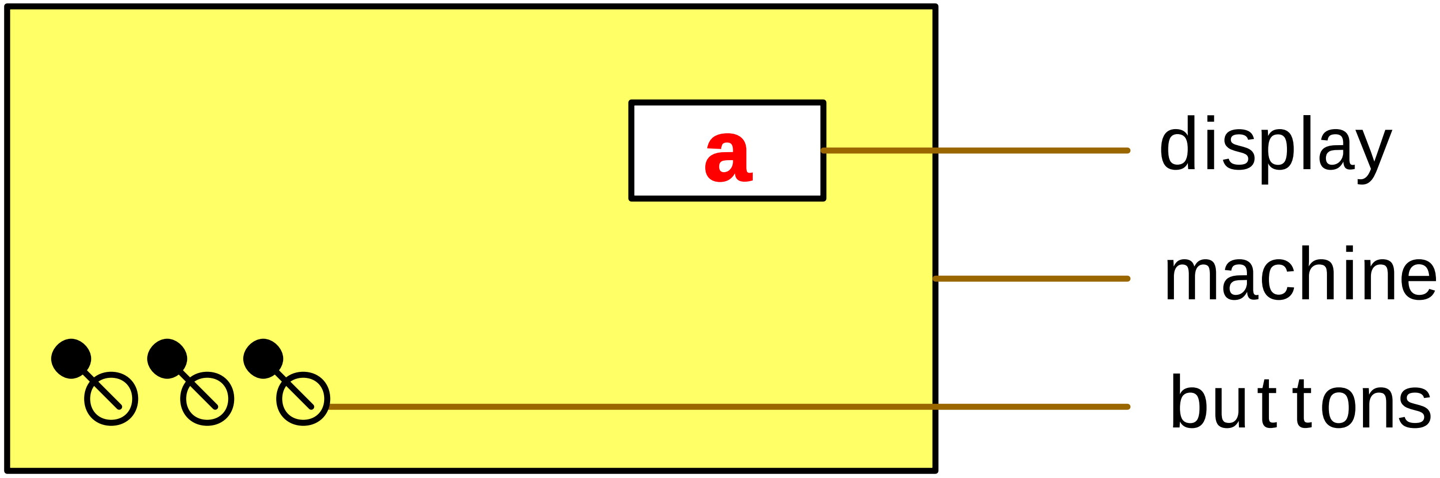


- an observer
  - records what s/he sees (over time) + buttons

- define **$Obs_M(P)$:**
  - **observations** of P

  = what observer records, if LTS P is inside M

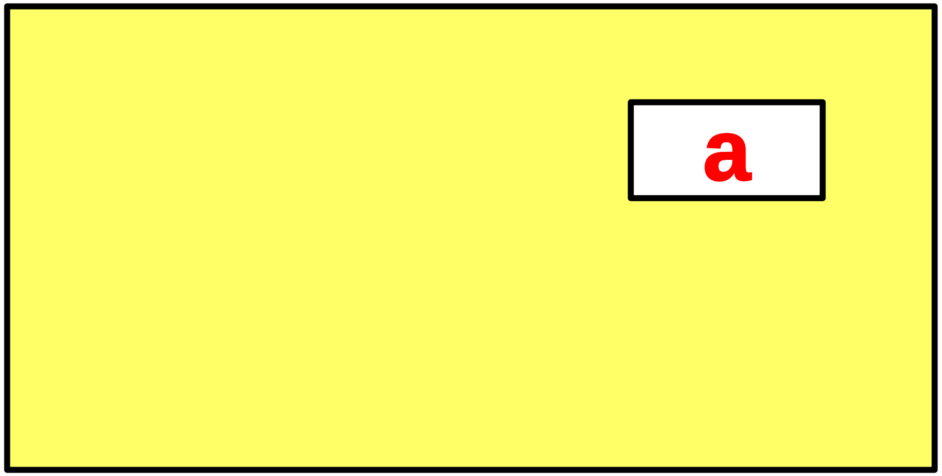
Model for testing scenarios



- processes (LTSs) with same observations are deemed to be equivalent.
- characterization results:

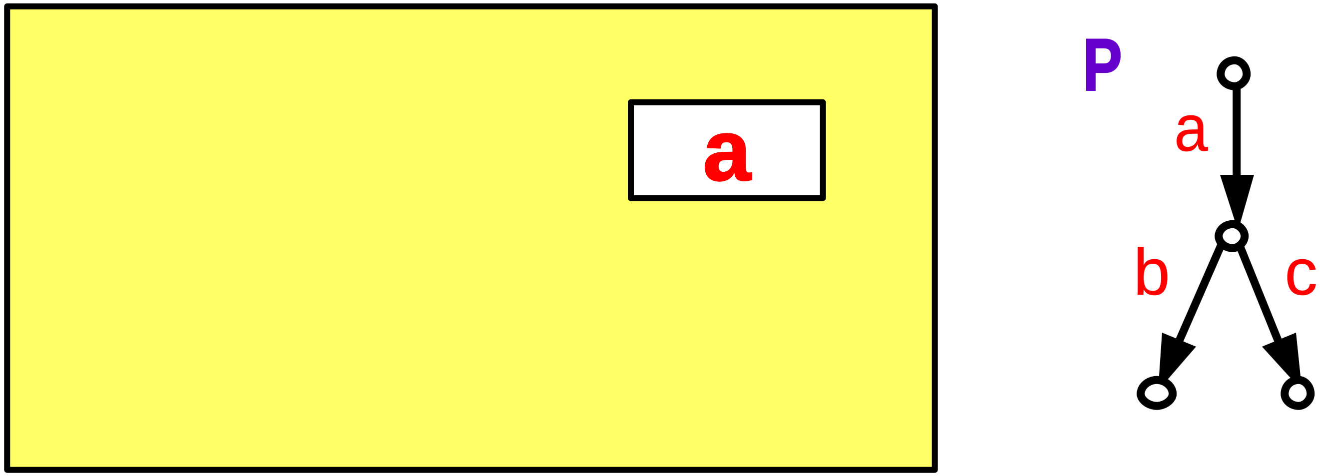
  $Obs_{M}(P) = Obs_{M}(Q)$ iff P ´ Q
- ´ does not distinguish too much/too little

Trace Machine (TM)



- no buttons for interaction
- display shows current action

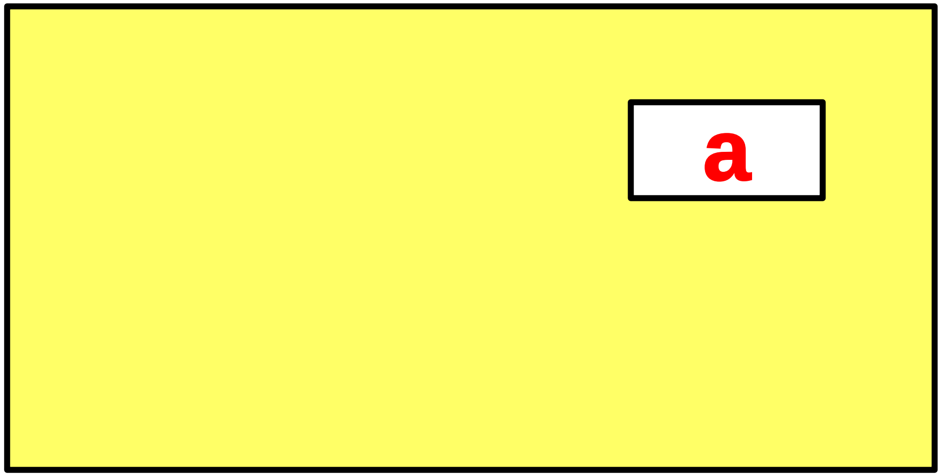
Trace Machine (TM)



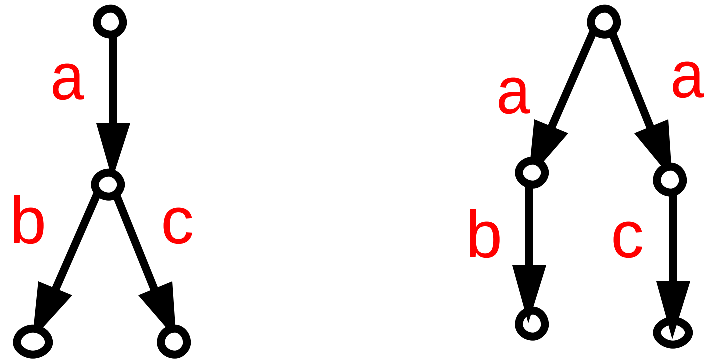
- no buttons for interaction
- display shows current action
- with **P** inside M, an observer sees either of

  ε, a, ab, ac
- $Obs_{TM}(P)$ = traces of P
- testing scenario for trace (language) equivalence

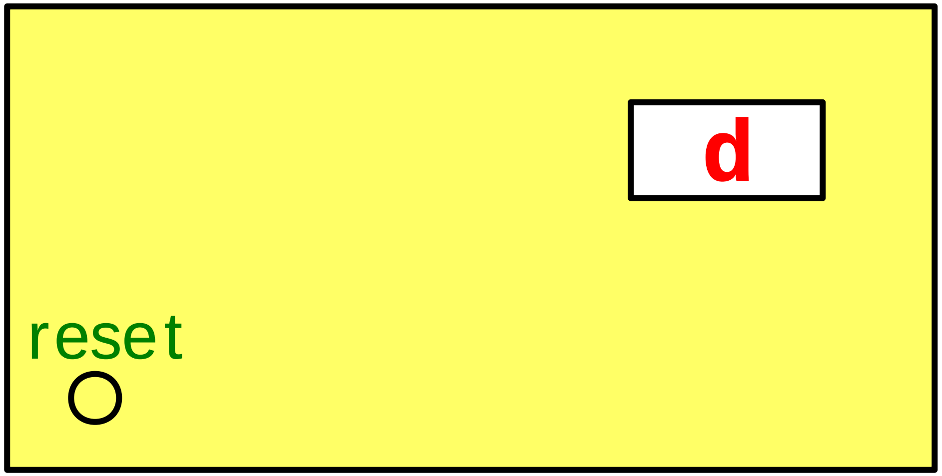
Trace Machine (TM)

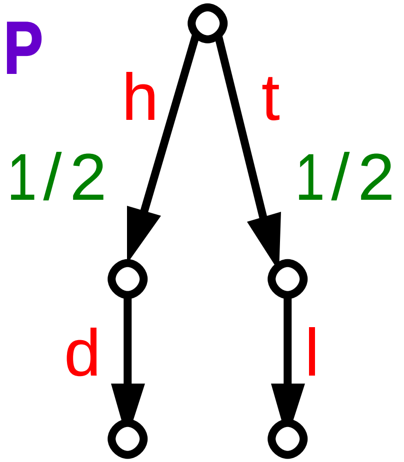


- no distinguishing observation between



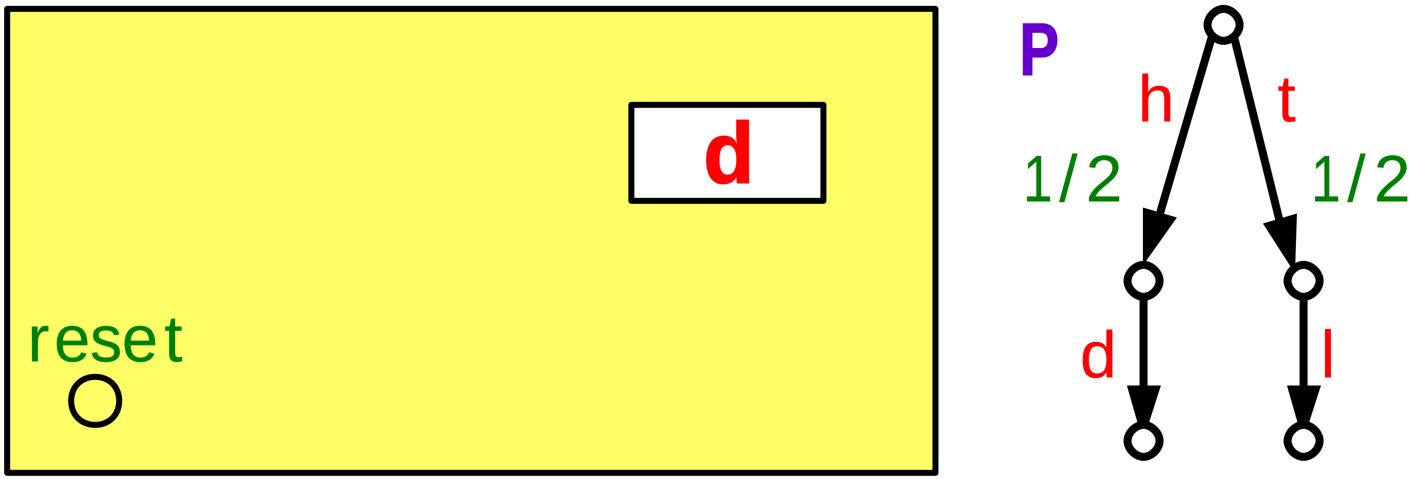
Trace Distribution Machine (TDM)





- reset button: start over
- repeat experiments
- each experiment yields trace of same length k
- observe frequencies of traces

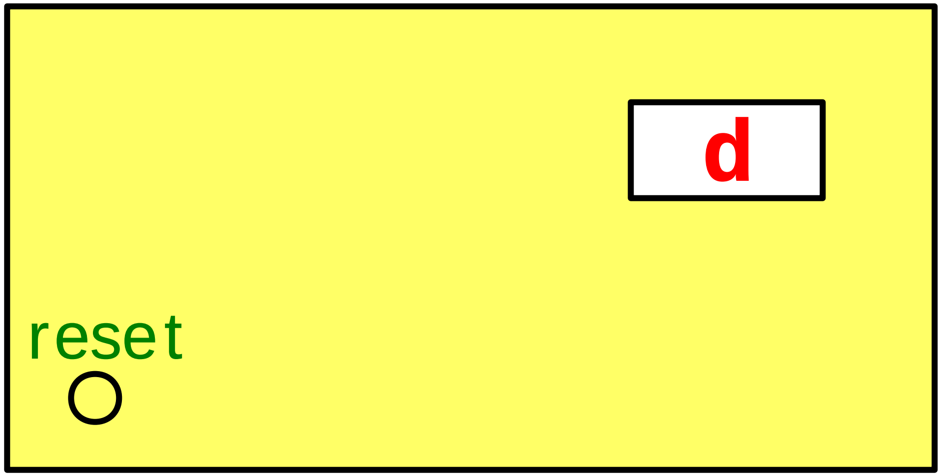
Trace Distribution Machine (TDM)

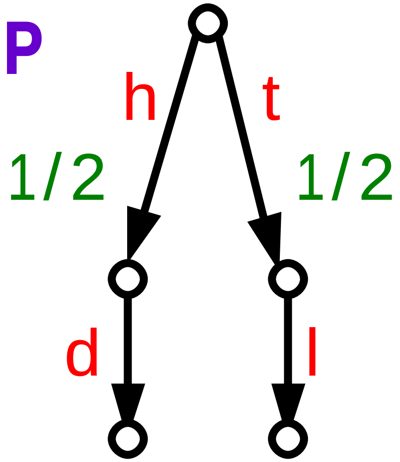


9 experiments, length 2

tl
hd
hd
tl
hd
hd
hd
tl
tl

Trace Distribution Machine (TDM)





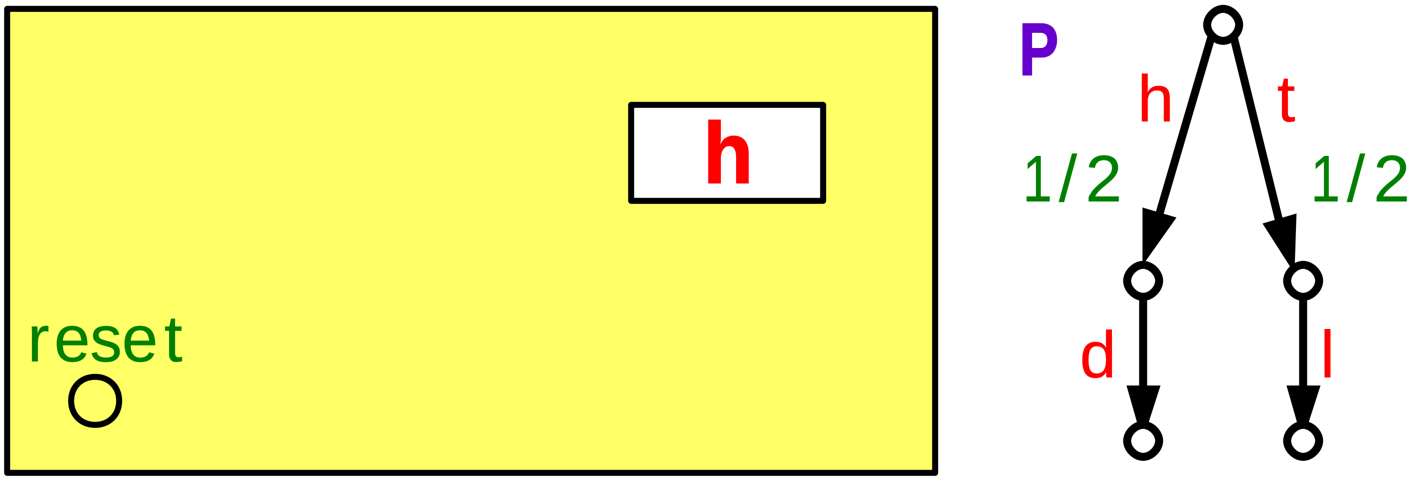
9 experiments, length 2

tl
hd
hd
tl
hd
hd
hd
tl
tl

frequencies

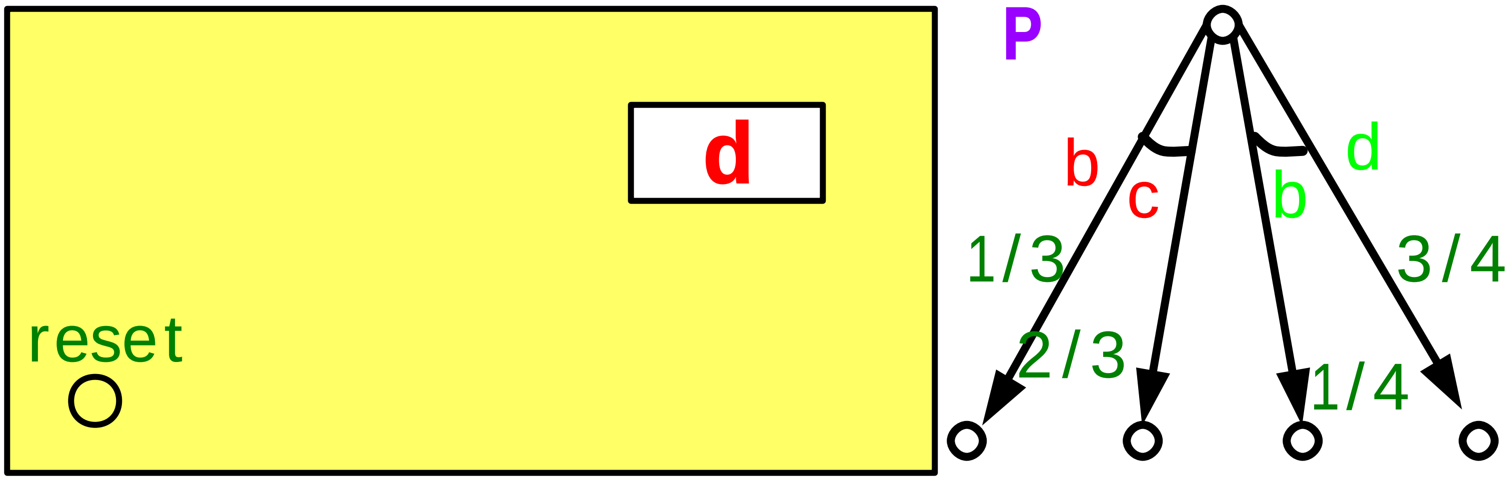
hd	4
tl	5
other	0

Trace Distribution Machine (TDM)



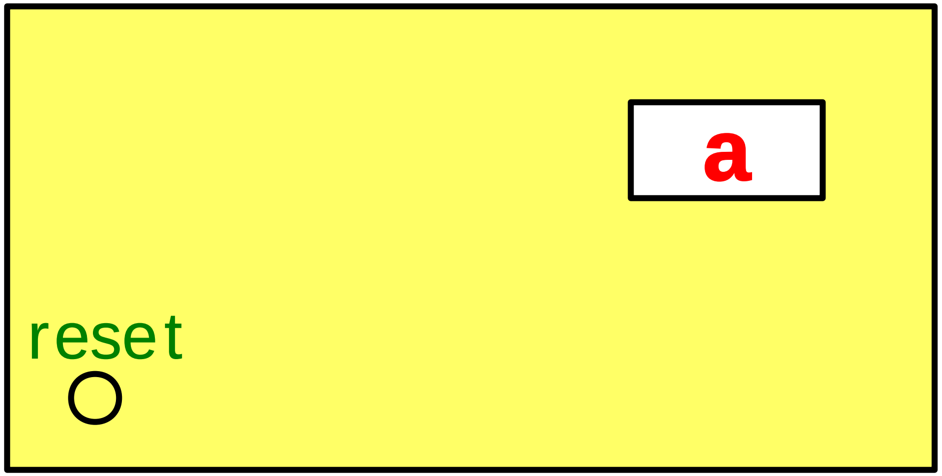
- with many experiments: #hd ¼ # tl
- use statistics: (m=100, k = 2)
  - hd,hd,hd,...,hd 2 Obs(P) freqs too unlikely to be an obsv
  - hd,tl,tl,hd,...tl,hd 2 Obs(P) freqs likely, is an observ of P

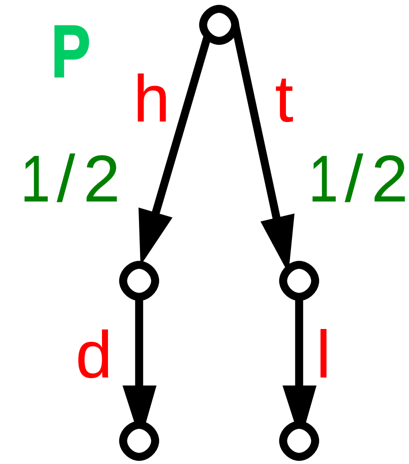
Trace Distribution Machine (TDM)



- nondeterministic choice
- choose one transition probabilistically
- in large outcomes: ½ #c + 1/3 #d ¼ #b
- use statistics:
  - b,b,b,....b 2 Obs(P) freqs too unlikely to be an obs
  - b,d,c,b,b,b,c,... 2 Obs(P) freqs likely, is an observ of P

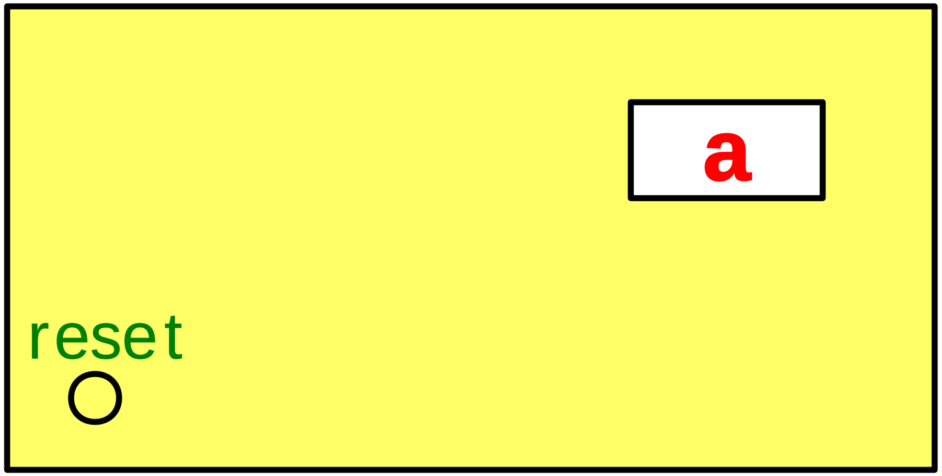
Observations TDM

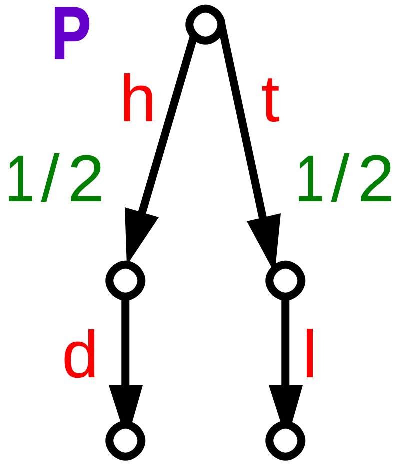




- Obs$_{TDM}$(P) =
  { σ | σ is likely to be produced by P}

Observations TDM



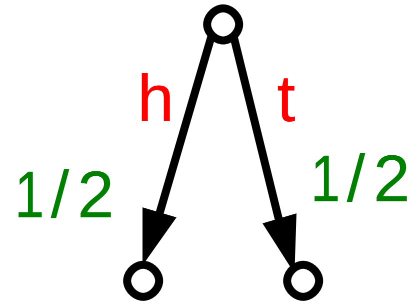


- perform m experiments (m resets)
- wlog: each experiment: trace of length k
- sample $\sigma 2 (Act^k)^m$
- Obs(P) =

  $\{\sigma 2 (Act^k)^m \mid \sigma$ is likely to be produced by P$\}$
- **what is likely?** use hypothesis testing

What outcomes are likely?

- I have a sequence σ = h,t,t,t,h,t,... 2 $\{h,t\}^{100}$
- I claim: generated σ with automaton P.
- do you believe me
  - if σ contains 15 h's?
  - if σ contains 42 h's?



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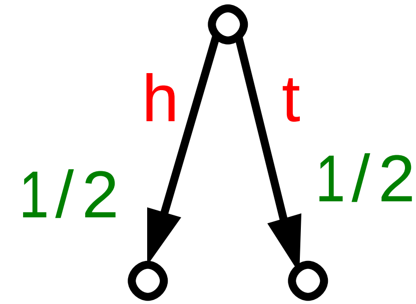
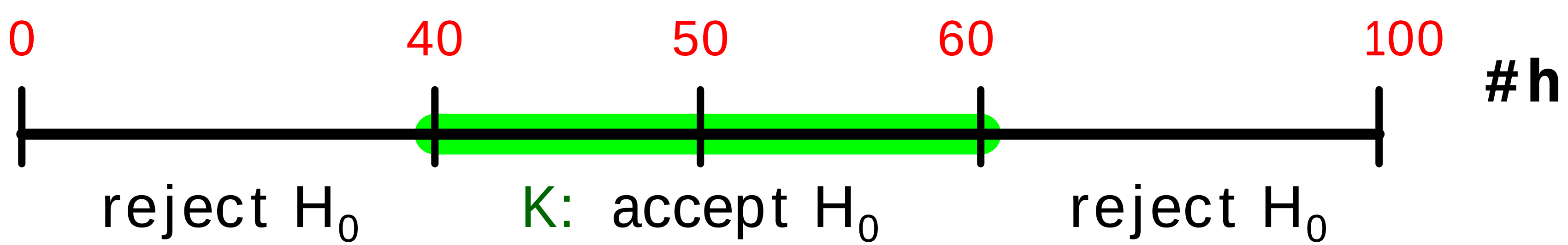


- use hypothesis testing:
- fix confidence level α 2 (0,1)
- H_0 null hypothesis = σ is generated by P



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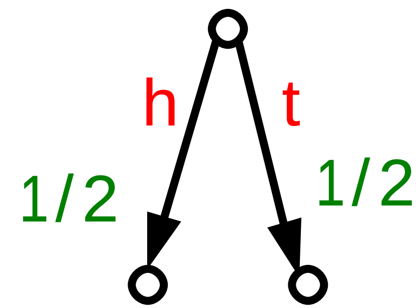
  - if σ contains 42 h's? YES



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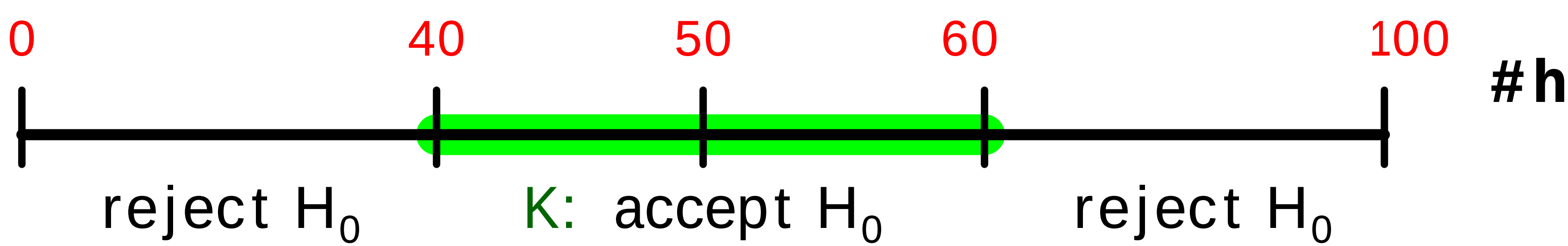

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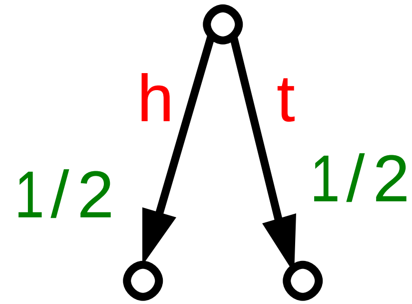
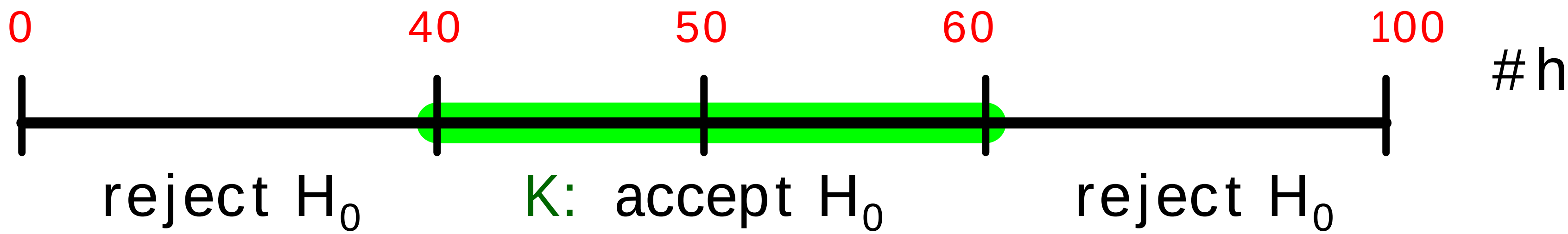


- use hypothesis testing:
- fix confidence level α 2 (0,1)
- H_0 null hypothesis = σ is generated by P

- $P_{H0}[K] > 1-\alpha$: prob on false rejection · α

  P : $_{H0}$[K] minimal : prob on false acceptance minimal

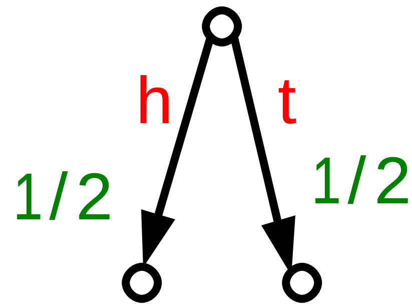


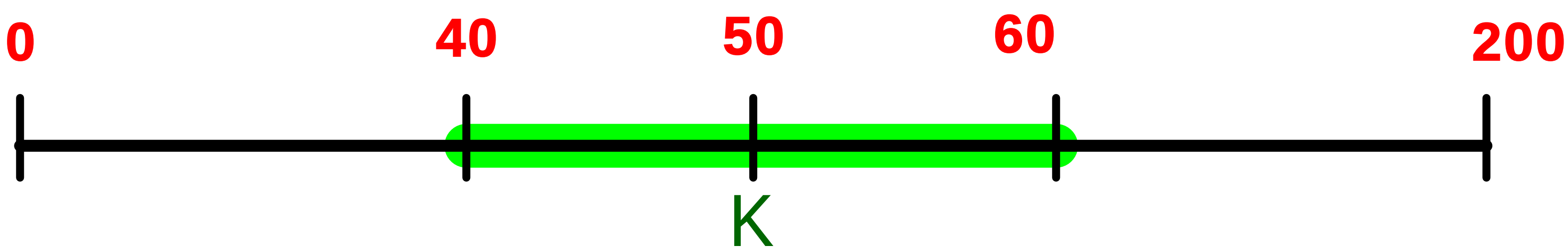
Observations a =0.05

- Obs(P) = {$\sigma 2\ (Act^k)^m$ | accept H_0 for σ, }
- for $k = 1$ and $m = 100$,

  $\sigma 2\ (Act)^{100}$ is an observation iff

  $40 \cdot freq_\sigma(h) \cdot 60$





Observations a =0.05

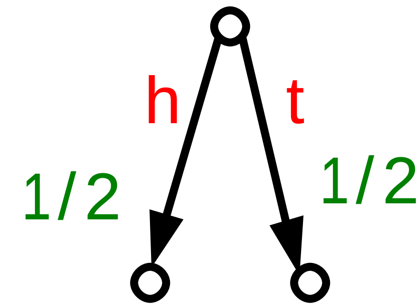
- Obs(P) = {σ2 $(Act^k)^m$ | σ is likely to be produced by **P**}
- for k = 1 and m = 100,

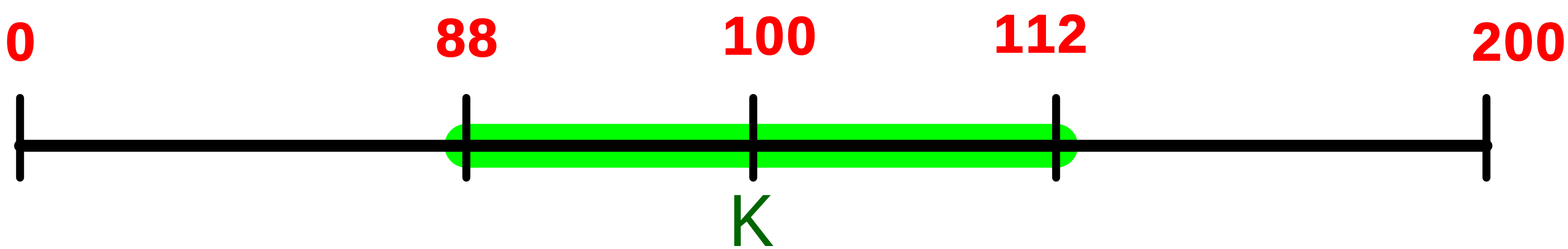
  σ2 $(Act)^{100}$ is an observation iff

  40 · $freq_\sigma$ (hd) · 60
- for k = 1 and m = 200

  σ2 $(Act)^{200}$ is an observation iff

  88 · $freq_\sigma$ (h) · 112
- etc....



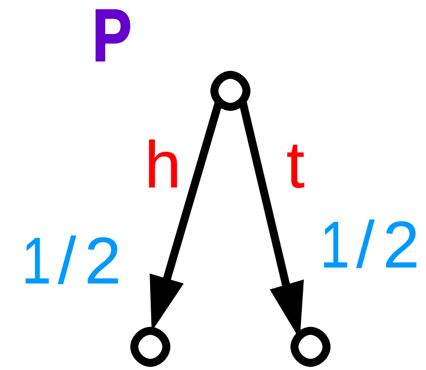


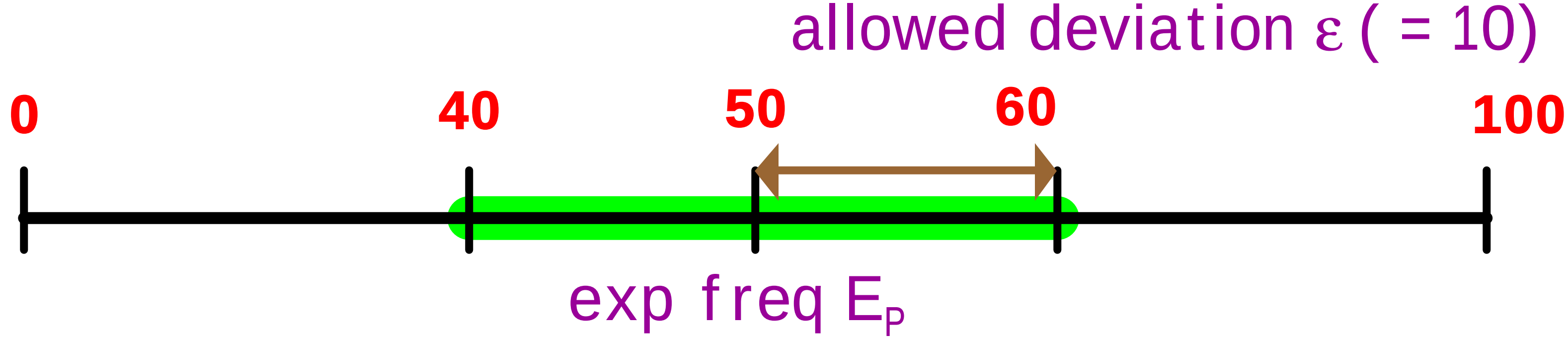
Observations a =0.05

- Obs(P) = {σ2 $(Act^k)^m$ | σ is likely to be produced by **P**}
- for k = 1 and m = 99

  σ2 $(Act)^{100}$ is an observation iff

  40 · $freq_\sigma$ (hd) · 60





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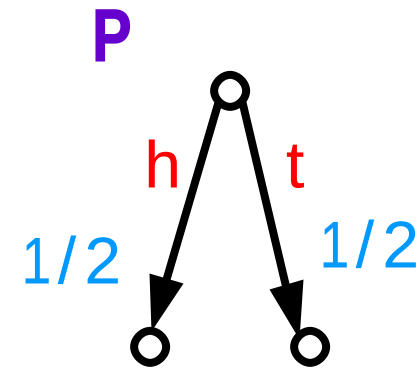
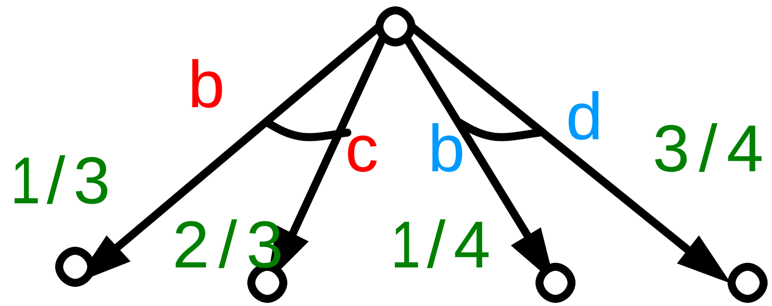


- K = $sphere_\varepsilon$ (E_P)
  = points within distance ε from exp val E_P
- ε is minimal with P[K] > 1-α

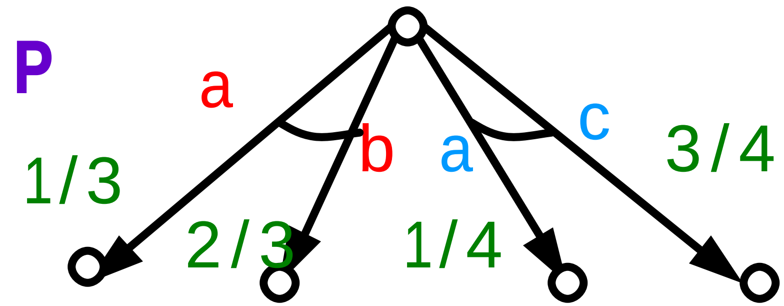
With nondeterminism



- σ = b,c,c,d,b,d,....,c 2 Obs(P) ??

- to compute expected frequencies and K,
  resolve notdet first
  - what is expected freq of b ?

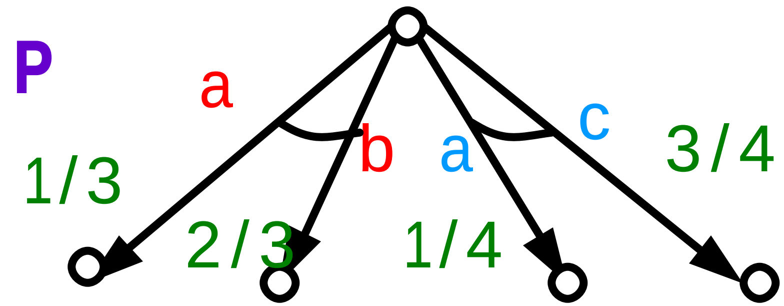
With nondeterminism



- σ = b,c,c,d,b,d,....,c 2 Obs(P) ??

- if we fix scheduler sequence: p_1, p_2, p_3... p_{100}
  - p_i = **P**[take left trans in experiment i]
  - 1 - p_i = **P**[take right trans in experiment i]

With nondeterminism



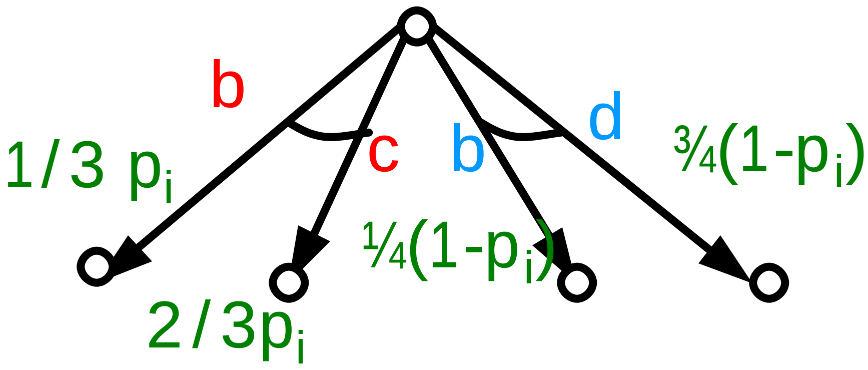
- σ = b,c,c,d,b,d,....,c 2 Obs(P) ??

- if we fix adversaries: p_1, p_2, p_3... p_{100}
  - p_i = P[take left trans in experiment i]
  - $1 - p_i$ = P[take right trans in experiment i]

- critical section $K_{p1,...,p100}$
  - H_0: σ is generated by **P** under p_1, p_2, p_3... p_{100}

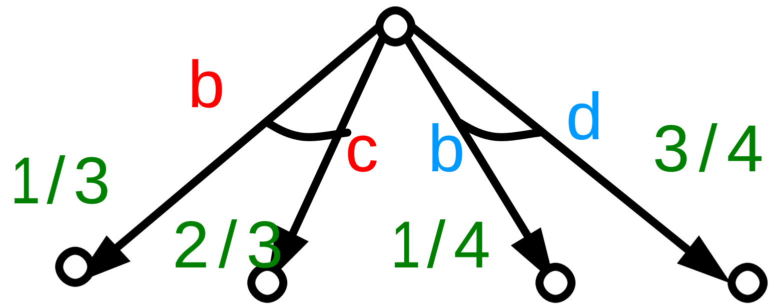
- σ 2 Obs(P) iff σ2 $K_{p1,...,p100}$ for some p_1, p_2, p_3... p_{100}

With nondeterminism



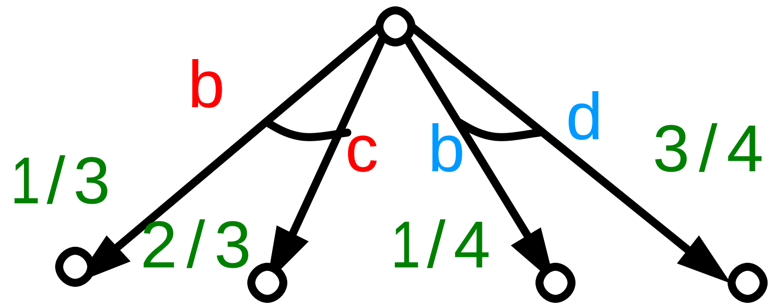
- **fix** $p_1, p_2, p_3 \ldots p_{100}$
- compute $P_{p1, p2, p3\ldots p100}[\sigma]$ for every σ
  - e.g $p_i = ½$, $\mathbf{P_{p1, p2, p3\ldots p100}}[c,c\ldots c] = (½ * 2/3)^{100}$
- **expected frequency** $E_{p1,\ldots,p100}$ for
  - $c = \sum_i 2/3\ p_i$
  - $d = \sum_i 3/4\ (1-p_i)$
  - $b = \sum_i 1/3\ p_i + ¼\ (1-p_i)$
- as before: critical section $K_{p1,\ldots,p100}$

With nondeterminism



- fix $p_1, p_2, p_3 \ldots p_{100}$
  - compute $P_{p1, p2, p3 \ldots p100}[\sigma]$ for every σ
- expected frequency E
- as before: critical section $K_{p1,\ldots,p100}$
  - H_0: σ is generated by P under $p_1, p_2, p_3 \ldots p_{100}$
  - allow observations to deviate $< \varepsilon$ from E
  - $K_{p1,\ldots,p100} = P_{p1, p2, p3 \ldots p100}[\text{sphere}_\varepsilon(E)]$
  - with ε minimal with $P_{p1, p2, p3 \ldots p100}[\text{sphere}_\varepsilon(E)] > \alpha$

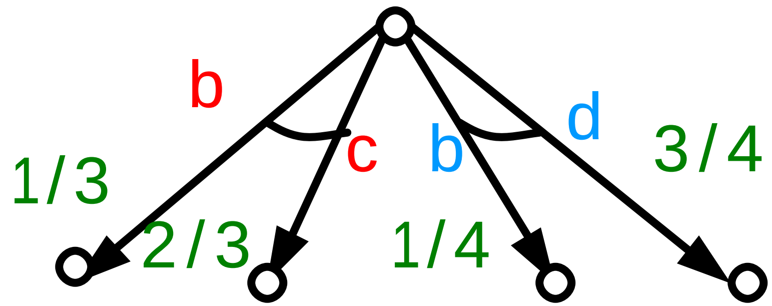
Observations



Observations for $k = 1$, $m = 100$.

- σ contains a,b only with $54 \cdot freq_{\sigma}(c) \cdot 78$
  - take $p_i = 1$ for all i
- σ contains b,d only with $62 \cdot freq_{\sigma}(d) \cdot 88$
  - take $p_i = 0$ for all i

Observations



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  - take $p_i = 0$ for all i

$m = 200$

- $61 \cdot freq_{\sigma}(c) \cdot 71$ and $70 \cdot freq_{\sigma}(d) \cdot 80$
  - $p_i = ½$ for all i
  - (these are not all observations; they form a sphere)

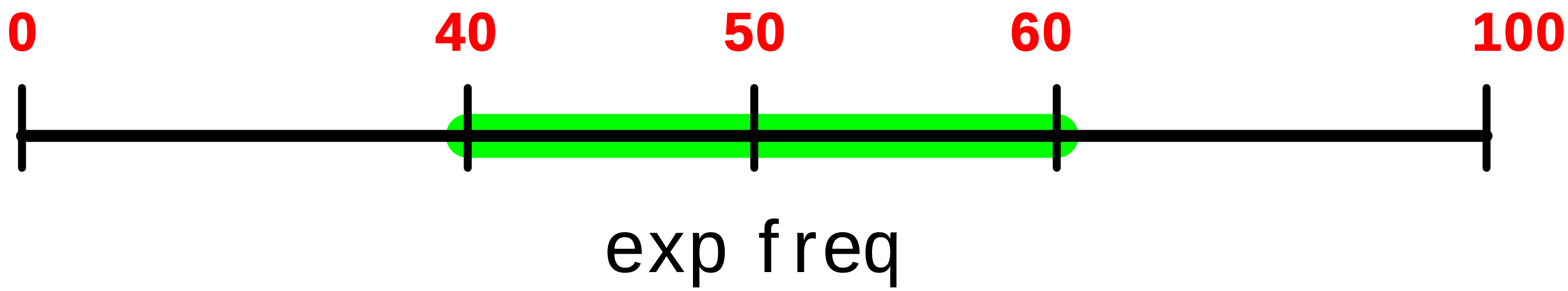
Main result

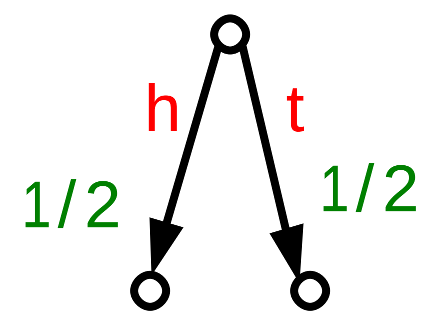
- TDM characterizes trace distr equiv:

$$Obs_{TDM}(P) = Obs_{TDM}(Q) \quad \text{iff} \quad trd(P) = trd(Q)$$

if P, Q are fin branching

- justifies trace distribution equivalence in an observational way


0
40
50
60
100
exp freq


h
t
1/2
1/2

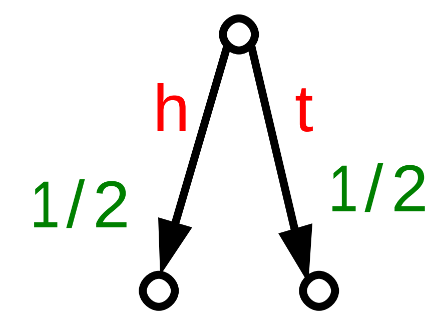
Observations a =0.05

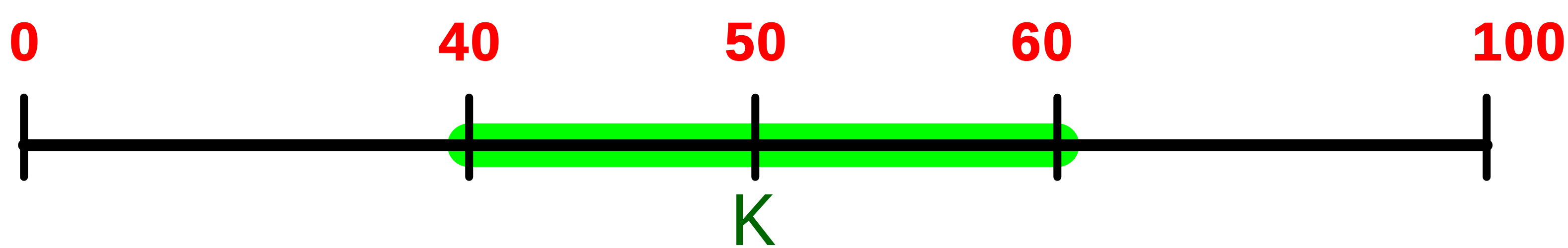
- Obs(P) = {σ2 $(Act^{k})^{m}$ | σ likely to be produced by **P**}
- Obs(P) = {σ2 $(Act^{k})^{m}$ | freq_σ in K}
-

- for k = 1 and m = 100,

  β2 $(Act)^{100}$ is an observation iff

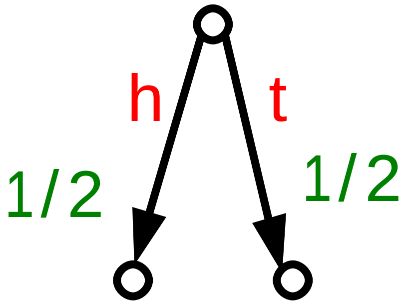
  40 · $freq_{\beta}$ (hd) · 60





Nondeterministic case

- \sigma = \beta_1 ,...\beta_m
- fixed adversaries
- take in
- expect_freq



- for \gamma\in Act^k,

  freq_\gamma(\beta)

freq \in \

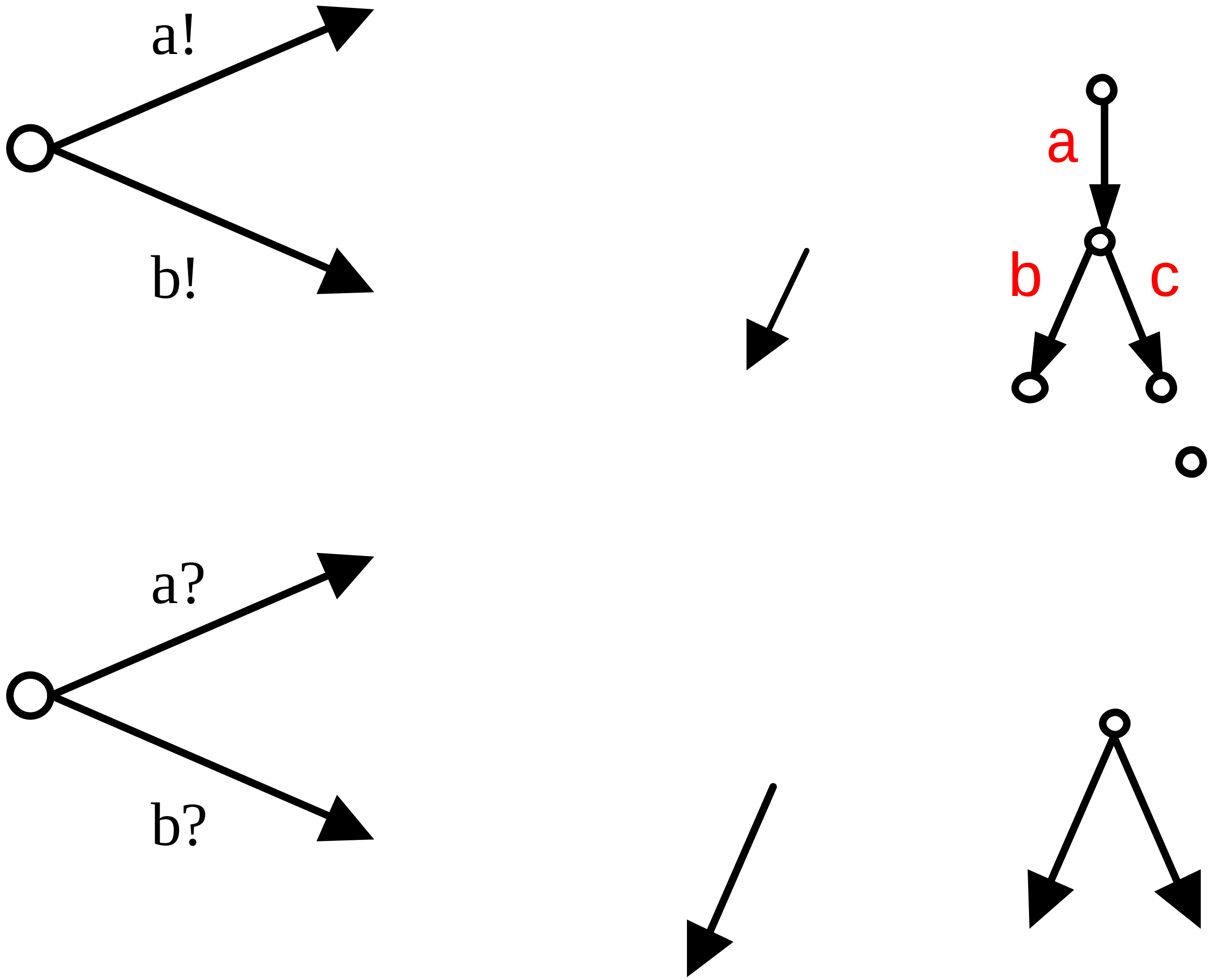
- we consider only frequency of traces in an outcome

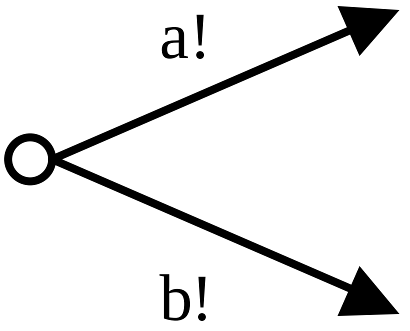
Main result

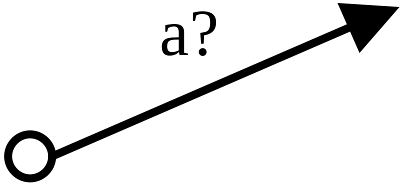
TDM characterizes trace distr equiv ´$_{TD}$

$$\text{Obs}_{\textbf{TDM}}(P) = \text{Obs}_{\textbf{TDM}}(Q) \quad \text{iff} \quad \text{trd}(P) = \text{trd}(Q)$$

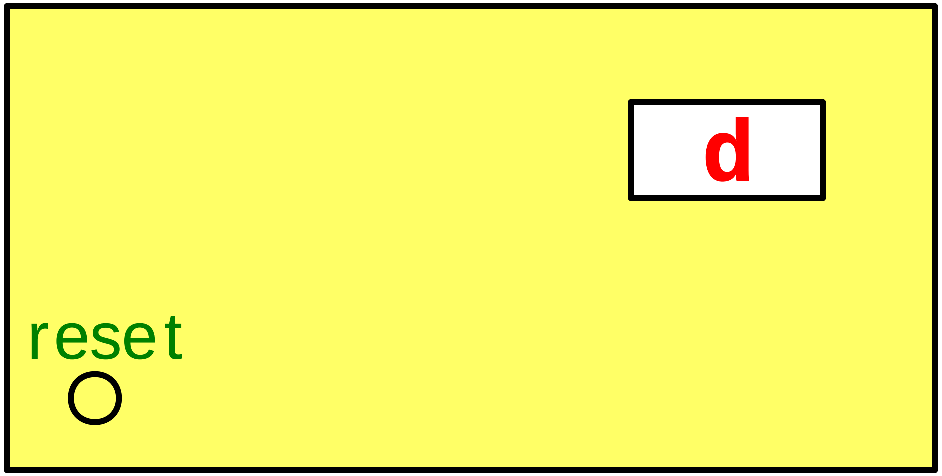
- "if" part is trivial, "only if"-part is hard.
  - find a distinguishing observation if P, Q have different trace distributions.
- IAP for P. Q fin branching
  - P, Q have the same infinite trace distrs iff
    P, Q have the same finite trace distrs
- the set of trace distrs is a polyhedron
- Law of large numbers
  - for random vars with different distributions

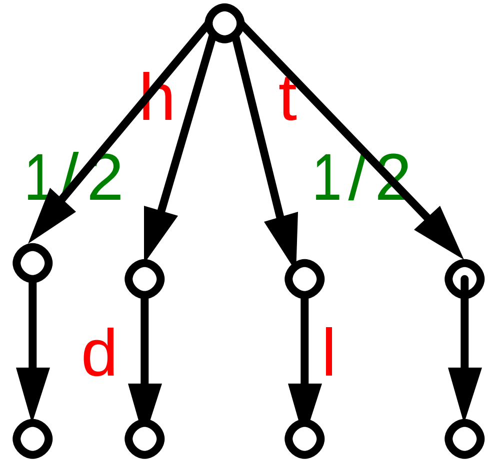

a!
b!
a?
b?
a
b
c


a!
b!


a?

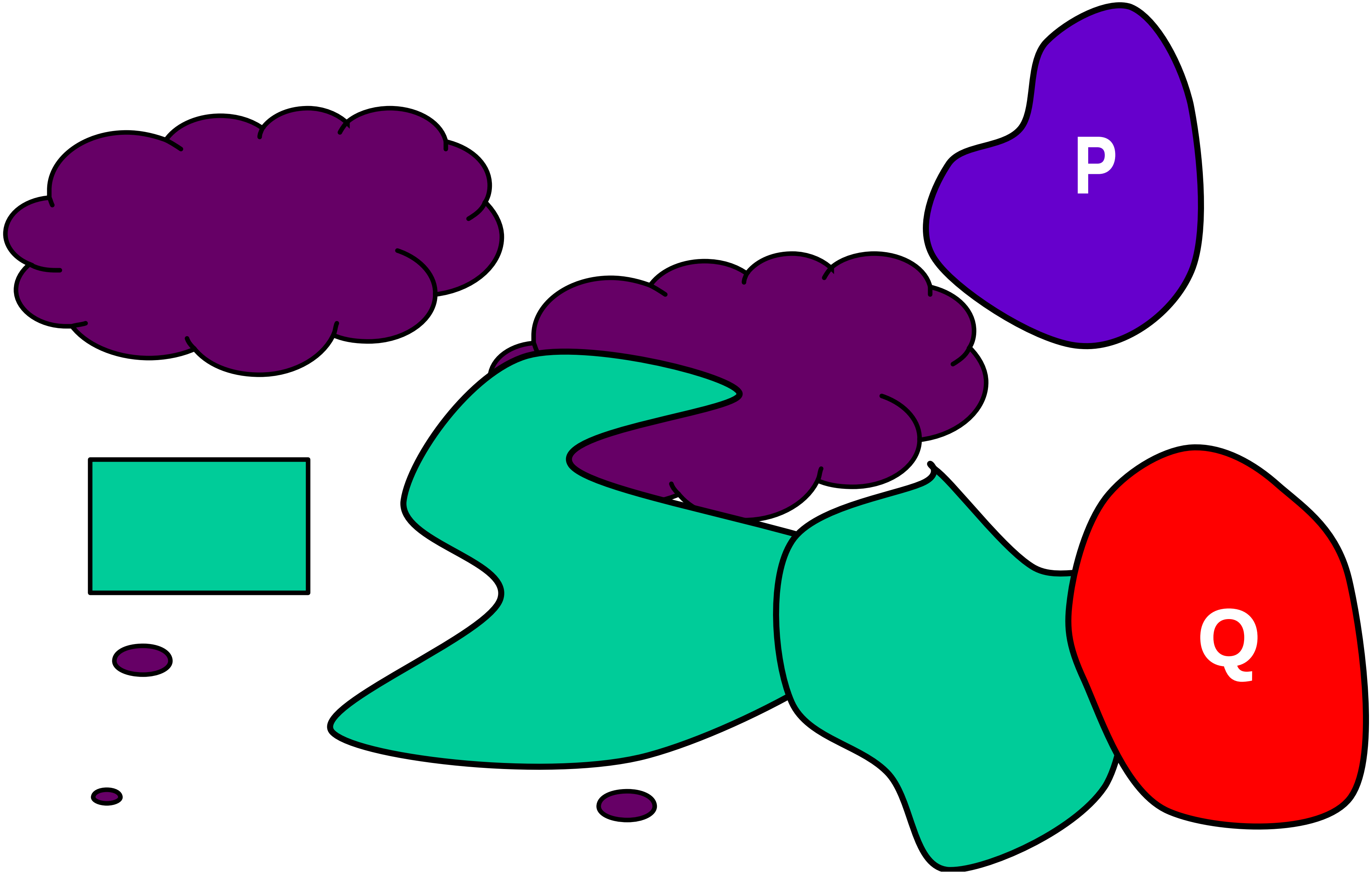
Trace Distribution Machine (TDM)



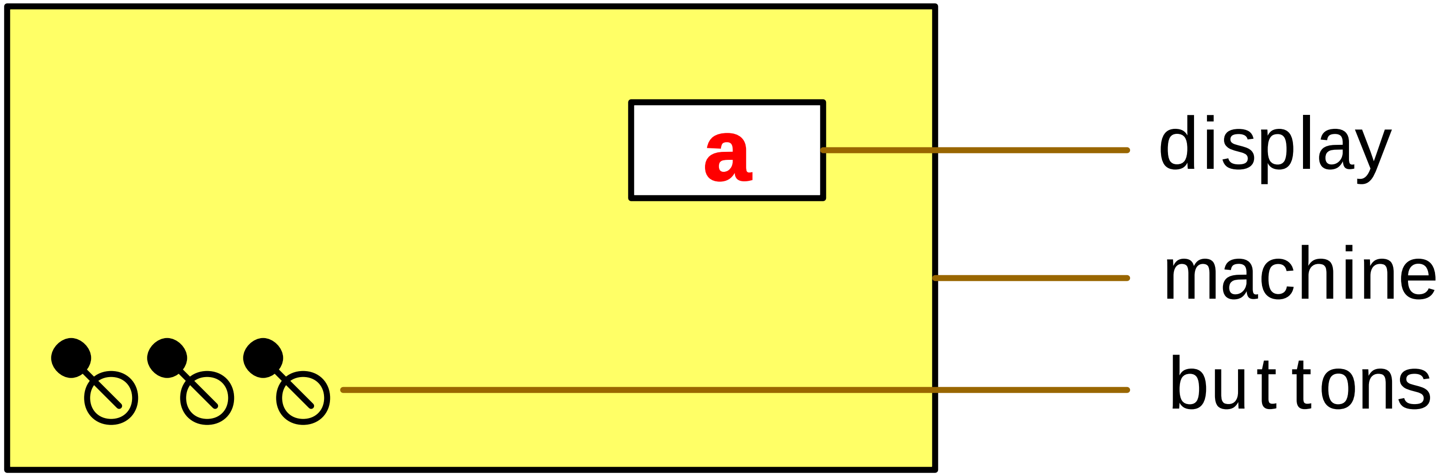


- reset button: start over
- repeat experiments: yields *sequence of traces*
- in large outcomes: #hd ¼ # tl
- use statistics:
  - hd,hd,hd,...,hd 2 Obs(P) too unlikely
  - hd,tl,tl,hd,...tl,hd 2 Obs(P) likely

Process equivalences



Testing scenario's



- a black box with display and buttons
- inside: process described by LTS **P**
- display: current action
- what do we see (over time)? $Obs_M(P)$
- P, Q are deemed equivalent iff $Obs_M(Q) = Obs_M(Q)$
- desired characterization:

Observations a =0.05

- Obs(P) = {σ2 $(Act^k)^m$ | σ is likely to be produced by **P**}
- for k = 1 and m = 99,
- expectation E = (33,33,33)
- Obs(P) = {σ2 $(Act)^{99}$ | | σ – E | < 15}