Basic Math Formulas

Arithmetic operations (ab means a×b)

\[
\begin{array}{|c|c|c|c|}
\hline
a(b + c) &= ab + ac \\
\hline
\frac{a + b}{c} &= \frac{a}{c} + \frac{b}{c} \\
\hline
\frac{a}{b} \times \frac{c}{d} &= \frac{ab}{bd} \\
\hline
\end{array}
\]

Logarithm: \( \log_a x = y \Leftrightarrow a^y = x \)

\[
\begin{align*}
\log_a (xy) &= \log_a (x) + \log_a (y) \\
\log_a (x/y) &= \log_a (x) - \log_a (y) \\
\log_a (x^r) &= r \log_a (x)
\end{align*}
\]

Factors: \((x + a)(x + b) = x^2 + (a + b)x + ab \)

\[
(x + a)(x - a) = x^2 - a^2 \\
ax^2 + bx + c = 0 \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Absolute Value (for \(a > 0\)): \[ |x| = a \Rightarrow x = a \text{ or } x = -a \]

Right triangle

Pythagorean Theorem: \(a^2 + b^2 = c^2\)

\[
\begin{align*}
\sin \theta &= \frac{\text{opp}}{\text{hyp}} = \frac{b}{c}, \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{a}{c} \\
\tan \theta &= \frac{\sin \theta}{\cos \theta} = \frac{\text{opp}}{\text{adj}} = \frac{b}{a}
\end{align*}
\]

Important angles

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\theta & \text{rad} = \frac{\theta}{180} \pi & \sin \theta & \cos \theta & \tan \theta & \text{rad} = \frac{\theta}{180} \pi \\
\hline
0^\circ & 0 & 0 & 1 & 0 & 0 \pi \pi \\
30^\circ & \frac{\pi}{6} & \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{3}} & \frac{\pi}{3} \pi \\
45^\circ & \frac{\pi}{4} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 & \frac{\pi}{2} \pi \\
60^\circ & \frac{\pi}{3} & \frac{\sqrt{3}}{2} & \frac{1}{2} & \sqrt{3} & \frac{\pi}{2} \pi \\
90^\circ & \frac{\pi}{2} & 1 & 0 & -- & \pi \pi \\
\hline
\end{array}
\]

Trigonometric properties

\[
\begin{align*}
\sin^2(\theta) + \cos^2(\theta) &= 1 \\
\sin(-\theta) &= -\sin(\theta) \\
\cos(-\theta) &= \cos(\theta) \\
\tan(-\theta) &= -\tan(\theta) \\
\sin(\frac{\pi}{2} - \theta) &= \cos(\theta) \\
\cos(\frac{\pi}{2} - \theta) &= \sin(\theta) \\
\sin(\theta + y) &= \sin(\theta)\cos(y) + \cos(\theta)\sin(y) \\
\cos(\theta + y) &= \cos(\theta)\cos(y) - \sin(\theta)\sin(y) \\
\sin(2\theta) &= 2\sin(\theta)\cos(\theta) \\
\cos(2\theta) &= 1 - 2\sin^2(\theta) \\
\tan(\theta) &= \frac{\sin(\theta)}{\cos(\theta)} \\
\end{align*}
\]
**Lines:** slope of a line through two points \((x_1, y_1)\) and \((x_2, y_2)\) is \( a = \frac{y_2 - y_1}{x_2 - x_1} \)

The slope (=a)/intercept(=b) form: \( y = ax + b \)

The slope/point \((x_1, y_1)\) form: \( y - y_1 = a(x - x_1) \)

Two lines \( y = a_1x + b_1 \) and \( y = a_2x + b_2 \) are - parallel if \( a_1 = a_2 \) and

- perpendicular if \( a_2 = -\frac{1}{a_1} \)

The distance between two points \( P_1(x_1, y_1) \) and \( P_2(x_2, y_2) \): \( |P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)

**Parabolas** \( y = ax^2 + bx + c \) \((a \neq 0)\) or \( y = a(x - d)^2 + e \) → extreme point \((d, e)\)

\( a > 0 \): parabola opens upward. \( a < 0 \): parabola opens downward.

y-intercept: \( y = c \)

x-intercept(s): solutions of \( ax^2 + bx + c = 0 \).

\( D = b^2 - 4ac > 0 \Rightarrow 2 \) x-intercepts, \( D = 0 \Rightarrow 1 \) and \( D < 0 \Rightarrow 0 \) x-intercepts.

Interval where \( y < 0 \): solve \( ax^2 + bx + c < 0 \)

**Functions** – Properties of a function \( f(x) \) and its graph \( y = f(x) \)

1. **\( D_f = \text{Domain of } f \):** all permitted values of \( x \).
   **Range of \( f \):** all possible values of \( f(x) \) \((x \text{ in } D_f)\)

2. **Vertical line test for a function \( f \):** \( x = a \) intersects the graph of \( f \) at most once

3. *Even* function: \( f(-x) = f(x) \). The graph can be reflected about the y-axis.
   *Odd* function: \( f(-x) = -f(x) \). The graph of the function can be reflected about the origin.

4. **Increasing** function: \( x_2 > x_1 \Rightarrow f(x_2) > f(x_1) \)
   **Decreasing** : \( x_2 > x_1 \Rightarrow f(x_2) < f(x_1) \)

5. **Composite function:** \( f \circ g(x) = f(g(x)) \) (“apply \( f \) after applying \( g \) to \( x \)”)

6. **Periodic function:** \( f \) has a period \( c \) if \( c \) is the smallest number such that \( f(x + c) = f(x) \).

7. **One-to-one** function: \( x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) \) “a horizontal line intersects at most once”

8. **The inverse \( f^{-1} \) of a one-to-one function \( f \):** \( f^{-1}(y) = x \ \Leftrightarrow \ f(x) = y \)

   *Find the inverse by:* 1. Checking if \( f \) is one-to-one 2. solving \( x \) from \( y = f(x) \) and
   3. Interchanging \( x \) and \( y \).

**Transformation of functions:**

1. **Shifting of a graph** \((c > 0)\)
   - \( y = f(x) + c \): shift \( y = f(x) \) \( c \) units upward
   - \( y = f(x) - c \): shift \( c \) units downward
   - \( y = f(x - c) \): shift \( c \) units to the right
   - \( y = f(x + c) \): shift \( c \) units to the left

2. **Stretching and reflecting** of a graph (for \( c > 1)\)
   - \( y \ = cf(x) \): stretch the graph \( y = f(x) \) vertically by a factor \( c \)
   - \( y = \frac{1}{c} f(x) \): compress vertically by factor \( c \)
   - \( y = f(cx) \): compress horizontally by factor \( c \)
   - \( y = f(x/c) \): stretch horizontally by factor \( c \)
• \( y = -f(x) \): reflect about the \( x \)-axis
• \( y = f(-x) \): reflect about the \( y \)-axis

3. **Graphing the inverse function** \( f^{-1} \): *Reflect the graph \( y = f(x) \) about the line \( y = x \).*

**Functions - Important functions**

**Polynomial function** of degree \( n \in \mathbb{N} \):

\[
y = a_n x^n + \ldots + a_1 x + a_0 \quad \text{if} \quad a_n \neq 0
\]

\( n = 0 \) constant function: the graph is a horizontal line \( y = a_0 \)  e.g. \( y = -2 \rightarrow \)

\( n = 1 \) linear function: line \( y = a_1 x + a_0 \)  ← e.g: \( y = -x + 1 \)

\( n = 2 \) quadratic function: the graph is a parabola \( y = a_2 x^2 + a_1 x + a_0 \)

\( \text{e.g.} \quad p(x) = -2x^2 + \frac{5}{2} x + \sqrt{3} \)

\( n = 3 \): cubic function, e.g.:

\[
p(x) = x^3 + \left( \sqrt{5} - 1 \right) x - 1
\]

**Power functions:** \( f(x) = x^a \), \( a \in \mathbb{R} \).
1. \( a \) is a positive integer
2. $a$ is a negative integer
e.g. $a = -1$ and $a = -2$:
\[ f(x) = x^{-1} = \frac{1}{x} \]
\[ f(x) = x^{-2} = \frac{1}{x^2} \]

3. $a = \frac{k}{n}$ is a rational number:
\[ f(x) = x^{\frac{1}{2}} = \sqrt{x} \]
\[ f(x) = x^{\frac{2}{3}} = \sqrt[3]{x^2} \]
Rational functions: \( f(x) = \frac{P(x)}{Q(x)} \),
for polynomials \( P(x) \) and \( Q(x) \).
Domain: \( Q(x) \neq 0 \)!
\[
f(x) = \frac{x^5 + 2x^3 - 6x + 1}{x^4 + 1}
\]

Algebraic functions: a function that is constructed
from polynomials by algebraic operations: addition, subtraction, multiplication, division or
taking roots. e.g:
\[
f(x) = \frac{\sqrt{x^2 + 1} - x + 3\sqrt[3]{x^5 - 3x^2 + 4}}{(x+4)^2}
\]

Exponential functions \( f(x) = a^x \), base \( a > 0 \)
Domain = \( \mathbb{R} \), Range = \((0, \infty)\) if \( a \neq 1 \)

Logarithmic functions \( f(x) = \log_a (x) \), for \( a = e \): \( f(x) = \ln (x) \)
Trigonometric Functions

\[
f(x) = \sin(x)
\]
\[
f(x) = \cos(x)
\]
\[
f(x) = \tan(x)
\]

<table>
<thead>
<tr>
<th>Odd function</th>
<th>Even function</th>
<th>Odd function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period: (2\pi)</td>
<td>Period: (2\pi)</td>
<td>Period: (\pi)</td>
</tr>
<tr>
<td>Range: ([-1, 1])</td>
<td>Range: ([-1, 1])</td>
<td>Range: (\mathbb{R})</td>
</tr>
</tbody>
</table>

Cosecant , secant and cotangent function

\[
f(x) = \csc x = \frac{1}{\sin x}
\]
\[
f(x) = \sec x = \frac{1}{\cos x}
\]
\[
f(x) = \cot x = \frac{1}{\tan x}
\]

Note: \(\sin^{-1} x \neq \frac{1}{\sin x}\)

(\(\sin^{-1} x\) is the inverse of \(\sin x\))

\(f(x) = \tan(x)\) has period \(\pi\):
\(f(x) = \tan(x)\) is one-to-one
if \(D_f = (-\frac{\pi}{2}, \frac{\pi}{2})\):
Then the inverse function exists on \(D_f\):
\(f^{-1}(x) = \tan^{-1}(x)\)
The inverse function of \( f(x) = \tan(x) \) is given by \( f^{-1}(x) \).
**Exercises Calculus: Basic Math and Functions**

Some exercises to improve your basic math skills (James Stewart, Appendices A, B and D)

| A1 | Rewrite without absolute value: a. $\sqrt{5} - 5$ b. $|x + 1|$ c. $|x^2 + 1|$ |
| A2 | Solve the inequality and give the solution set as an interval: a. $2x + 7 > 3$ b. $1 - x \leq 2$ c. $2x + 1 < 5x - 8$ d. $4x < 2x + 1 \leq 3x + 2$ e. $x^3 - x^2 \leq 0$ f. $\frac{1}{x} < 4$ |
| A3 | Solve: a. $|x| = 3$ b. $|x + 5| \geq 2$ |
| A4 | Solve for negative constants $a$, $b$ and $c$: $ax + b < c$ |
| A5 | Solve, if possible, by factorizing: a. $x^2 - 10x + 25 = 0$ b. $x^2 = 8x + 9$ c. $9x^2 + 9x + 1 = 0$ d. $x^2 = 3x - 10$ |
| A6 | If a ball is kicked upward from the top of a building 128 ft high with initial velocity 16 ft/s, then the height $h$ above the ground $t$ seconds later will be $h = 128 + 16t - 16t^2$. During what time interval will the ball be at least 32 feet above ground? |

**B1** Find the distance between the points $(1, 1)$ and $(4, 5)$

**B2** Find the slope of the line through the points $P(-3, 3)$ and $Q(-1, -6)$

**B3** Find the equation of the line if a. Through $(2, -3)$ and slope 6 b. the x-intercept = 1 and y-intercept = -3 c. Through (-1, -2), perpendicular to the line $2x + 5y = 8$

**B1** Find sin $\theta$, cos $\theta$ and tan $\theta$ if $\sin \theta = \frac{3}{5}$, $0 < \theta < \frac{\pi}{2}$

**B2** Find sin $\theta$ and tan $\theta$ if $\sin \theta = 3/5$, $0 < \theta < \frac{\pi}{2}$

**B3** Find $\sin \theta$, $\cos \theta$ and $\tan \theta$ for $\theta = \frac{3\pi}{4}$ b. $\frac{9\pi}{2}$ c. $\frac{5\pi}{6}$

**B4** Find the equation of the line if a. Through the points $P(2, 3)$ and $Q(-1, 0)$ b. Through $(1, 4)$, perpendicular to the line $2x + 5y = 8$

**D1** Find $\sin \theta$, $\cos \theta$ and $\tan \theta$ for $\theta = a. \frac{3\pi}{4}$ b. $\frac{9\pi}{2}$ c. $\frac{5\pi}{6}$

**D2** Find $\cos \theta$ and $\tan \theta$ if $\sin \theta = \frac{3}{5}$, $0 < \theta < \frac{\pi}{2}$

**D3** If in a right angle triangle the hypotenuse has length 10 and $\theta = 30^\circ$, compute the length of the adjacent side

**D4** Find all values of $x$ in $[0, 2\pi]$ satisfying: a. $2\cos x = 1$ b. $2 \sin^2 x = 1$ c. $\sin x < \frac{1}{2}$ d. $\sin(\frac{x}{2}) = \frac{1}{2}\sqrt{3}$

---

**Test your basic math skills** (compute the result yourself and choose the correct answer a, b, c or d)

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\frac{10}{3^2} = \frac{0.9}{1}$</td>
<td>$\frac{10}{\sqrt{3}} = \frac{10}{9}$</td>
<td>$\frac{9}{3} = 90$</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Simplify $\frac{4}{x^2}$</td>
<td>$2^{\frac{1}{2}} = \frac{21}{3}$</td>
<td></td>
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</tr>
<tr>
<td>3.</td>
<td>Simplify $\frac{\sqrt{3}}{x^3}$</td>
<td>$2^{\frac{1}{3}} = \frac{13}{3}$</td>
<td></td>
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</tr>
<tr>
<td>4.</td>
<td>$a + 3)^2 - a = 2x + 5$</td>
<td>$10a + 25 = 5$</td>
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</tr>
<tr>
<td>5.</td>
<td>$x^2 - x = \frac{x}{x^2 - 2x + 1} = \frac{1}{-x}$</td>
<td>$-2x - 1 = 2x - 1$</td>
<td></td>
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<tr>
<td>6.</td>
<td>$x + x - 1 = \frac{2x}{x^2 - 1} = \frac{2x}{x^2 - 2}$</td>
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<tr>
<td>7.</td>
<td>If $a = 1$ and $b = 2$, then $-(a^2b^2 - 2 - (a^2b)^2 = \frac{-26}{10} = 22$</td>
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<tr>
<td>8.</td>
<td>Solve $2x^2 + 6x - 9 \geq -x^2 + 3x + 3$</td>
<td>$x \leq 1$</td>
<td></td>
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<tr>
<td>9.</td>
<td>Solve $y$ from $\left{ \begin{array}{c} x + 3y = 8 \ x + 6y = 5 \end{array} \right.$</td>
<td>$-13/3 = 23/3$</td>
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</tr>
<tr>
<td>10.</td>
<td>Solve $\cos(2x) = \frac{1}{2}$ for $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$</td>
<td>$\frac{2\pi}{3}, \frac{5\pi}{3}, \frac{2\pi}{3}$</td>
<td></td>
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</tr>
<tr>
<td>11.</td>
<td>$2\sin x - 1 = 1 - \cos^2 x$</td>
<td>$2 + \ln(e - 1)$</td>
<td></td>
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</tr>
<tr>
<td>12.</td>
<td>$\sin(\frac{x}{2}) = \sin x$</td>
<td>$\cos x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td>The number of x-intercepts of $f(x) = x^3 + 6x^2 - 16$ is</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.</td>
<td>$\ln(e^x - e^{-x}) = \ln(3)$</td>
<td>$\ln(4.5) = 2 + \ln(e - 1)$</td>
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<tr>
<td>15.</td>
<td>The solution of the equation $e^{2x} = 9$ is</td>
<td>$\ln(3)$</td>
<td></td>
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<tr>
<td>16.</td>
<td>Which of the equalities is/are correct? (more than 1 answer is possible)</td>
<td>$\frac{1}{p} + \frac{1}{p} = \frac{1}{p^2}$</td>
<td></td>
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<tr>
<td>17.</td>
<td>Which of the equalities is/are correct for all $x \neq 0, 1, -1$?</td>
<td>$x = 1 + \frac{1}{x}$</td>
<td></td>
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<tr>
<td>18.</td>
<td>Which of the equalities is/are correct for $p &gt; 0$</td>
<td>$p^2 = \left(\frac{a}{b}\right)^2$</td>
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<tr>
<td>19.</td>
<td>Write $\frac{1}{a+b}$ as $\frac{a}{b}$ (where $a$ and $b$ are the smallest possible positive integers)</td>
<td>$\frac{a}{b}$</td>
<td></td>
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</tbody>
</table>
### Exercises for Functions and their properties

#### 1.1
Find the domain of the function
\[ a. \ f(x) = \frac{x+2}{x^2-1} \quad b. \ f(t) = \sqrt[3]{t-1} \]

#### 1.2
Find the domain and the range and sketch the graph of the function
\[ a. \ g(x) = \sqrt{x-5} \quad b. \ f(x) = \left\{ \begin{array}{ll} x^2+2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{array} \right. \]

#### 1.3
Determine whether \( f \) is even, odd or neither. If \( f \) is even or odd use symmetry to sketch its graph
\[ a. \ f(x) = x^2 \quad b. \ f(x) = x^3 + x \quad c. \ f(x) = x^3 - x \]

#### 1.4
Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential or logarithmic function:
\[ a. \ f(x) = \frac{3}{x^2} \quad b. \ g(x) = \sqrt{1-x^2} \quad c. \ h(x) = x^9 + x^4 \quad d. \ k(x) = \frac{x^2+1}{x^3+x} \]
\[ e. \ r(x) = \tan 2x \quad f. \ s(x) = \log_{10} x \]

#### 1.5
Suppose the graph of \( f \) is given (e.g. \( f(x) = x^2 \)). Write equations for the graph that are obtained from the graph \( y = f(x) \) as follows:
\[ a. \ \text{Shift 3 units upward} \quad b. \ \text{Shift 3 units downward} \quad c. \ \text{Shift 3 units to the right} \]
\[ d. \ \text{Shift 3 units to the left} \quad e. \ \text{Reflect about the x-axis} \quad f. \ \text{Reflect about the y-axis} \]
\[ g. \ \text{Stretch vertically by a factor of 3} \quad h. \ \text{Shrink vertically by a factor of 3} \]

#### 1.6
Given is the graph \( y = x^2 \). Sketch it and also
\[ a. \ y = f(x+2) \quad b. \ y = f(x)+2 \quad c. \ y = 2f(x) \quad d. \ y = \frac{1}{2}f(x)+3 \]

#### 1.7
Graph each function by starting with the graph of a “standard function” and then applying an appropriate transformation:
\[ a. \ y = -1/x \quad b. \ y = \cos (x/2) \quad c. \ y = \frac{1}{x-3} \quad d. \ y = 1 + 2x - x^2 \]

#### 1.8
Find \( f+g \), \( f-g \), \( fg \) and \( f/g \) if \( f(x) = x^3 + x \) and \( g(x) = 3x^2 - 1 \)

#### 1.9
Find \( f \circ g \), \( g \circ f \), \( f \circ f \) and \( g \circ g \) and state its domain if:
\[ a. \ f(x) = 2x^2 - x \quad g(x) = 3x + 2 \]
\[ b. \ f(x) = \sqrt{x-1} \quad g(x) = x^2 \]
\[ c. \ f(x) = \frac{1}{x} \quad g(x) = x^3 + x \]

#### 1.10
Find \( f \circ g \circ h \) and state its domain if \( f(x) = x - 1 \), \( g(x) = \sqrt{x} \) and \( h(x) = x - 1 \)

#### 1.11
Write an equation that defines the exponential function with base \( a > 0 \). State the domain and range if \( a = 1 \). Sketch the general shape of the exponential function for the cases:
\[ (1) \ a > 1 \quad (2) \ a = 1 \quad (3) \ 0 < a < 1 \]

#### 1.12
Use the general shapes of 1.11 to give a rough sketch of:
\[ a. \ y = 2^x + 1 \quad b. \ y = 3^{-x} \quad c. \ y = 3 - 2^x \]

#### 1.13
Find the exponential function \( f(x) = Ca^x \) whose graph is through the points (1,6) and (3,24)

#### Exercises inverse functions and logarithmic functions:

#### 1.14
Find the exact value of each expression:
\[ a. \ \log_2 (64) \quad b. \ \log_6 \left( \frac{1}{36} \right) \quad c. \ 2^\log_2(3) + \log_2(5) \quad d. \ e^{3\ln(2)} \]

#### 1.15
Express the given quantity as a single logarithm:
\[ a. \ 2\ln(4) - \ln(2) \quad b. \ \ln (x) + a \ln (y) - b \ln(z) \]

#### 1.16
Solve each equation for \( x \):
\[ a. \ e^x = 16 \quad b. \ \ln(x) = -1 \quad c. \ 2x - 5 = 3 \quad d. \ \ln(x) - \ln(x-1) = 1 \]

#### 1.17
a. What is a one-to-one function?
   b. How can you tell from the graph that the inverse function exists?

#### 1.18
Determine whether \( f(x) \) is one-to-one if
\[ a. \ f(x) = 7x - 3 \quad b. \ g(x) = |x| \]

#### 1.19
Find the formula of the inverse function if:
\[ a. \ f(x) = \frac{1+3x}{5-2x} \quad b. \ f(x) = \sqrt{2 + 5x} \quad c. \ f(x) = \ln(x+3) \]

#### 1.20
If a bacteria population starts with 100 bacteria and doubles every three hours then the number of
bacteria after \( t \) hours is \( n = f(t) = 100 \times 2^{t/3} \)
\[ a. \ \text{Find the inverse of this function and explain its meaning.} \]
\[ b. \ \text{When will the population reach 50000?} \]
Solutions to the extra basic math exercises:

| A | 1a. 5 - √5 | 1c. x^2 + 1 | 2c. x > 3 | 2f. x < 0 or x > 1/4 | 4. x > (c-b)/a | 5c. -√5 |
|   | 1b. x + 1 for x ≥ -1 | 2a. x > -2 | 2d. -1 ≤ x < 1/2 | 3a. x = ±3/2 | 5a. 5 | 5d. none (D = 9-40 < 0) |
|   | -x - 1 for x < 1 | 2b. x ≥ -1 | 2e. x ≤ 1 | 3b. x ≤ -7 or x ≥ -3 | 5b. 1, -9 | 6. [0, 3] or 0 ≤ t ≤ 3 (1 > 0!) |

| B | 1. 5 | 2. -9/2 | 3a. y = 6x - 5 | 3b. y = 3x - 3 | 3c. 5x - 2y + 1 = 0 |

| D | 1a. sin(3π/4) = √2, cos(3π/4) = -√2, tan(3π/4) = -1 | 2. cos θ = 4/5, tan θ = 1/4 | 4a. 3π/4, 5π/3 | 4c. 0 ≤ x ≤ π/6 |
|   | 1b. sin(9π/2) = 1, cos(9π/2) = 0, tan(9π/2): undefined | 4b. 1/4 π , 3π/4, 5π/4 | 3. 5π/3 | 4d. 7π/4, 4π/3 |
| 1c. √3, 3/√3, 1/√3 |

Answers to the test:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| d | b | c | a | d | d | b | c | c | a | d | c | b | a | c | b | d | b | c |

19. \( \frac{3}{16} \) 20. \( (\frac{mc}{100-c})^2 \) if \( \frac{mc}{100-c} > 0 \)

Solutions for the Function-exercises:

1.1 a. D = \( \{ x | x \neq \pm 1 \} \) b. D = \( (-\infty, \infty) \)

1.2 a. D = \( [5, \infty) \), R = \( [0, \infty) \) b. D = \( (-\infty, \infty) \), R = \( (-\infty, \infty) \)

1.3 a. Even (graph by reflecting about the y-axis) b. Neither c. Odd (graph by reflecting about the origin) d. Rational e. Trigonometric f. Polynomial, degree 9


1.5 a. \( y = f(x) + 3 \) b. \( y = f(x) - 3 \) c. \( y = f(x - 3) \) d. \( y = f(x + 3) \) e. \( y = f(x) \)

1.6 a. \( f + g(x) = x^3 + 3x^2 + x - 1 \), \( f - g(x) = x^3 - 3x^2 + x + 1 \), \( f \cdot g(x) = (x^3 + x)(3x^2 - 1) \) and \( f/g(x) = \frac{x^3+x}{3x^2-1} \)

1.7 a. Start with \( y = 1/x \) and reflect it about the x-axis b. Start with \( y = \cos x \) and stretch horizontally by a factor of 2 c. Start with \( y = 1/x \) and shift it 2 units to the right d. \( y = 1 + 2x - x^2 = 2 - (x - 1)^2 \) e. Start with \( y = x^2 \), shift it 1 to the right, reflect about the x-axis and shift it 2 units upward.

1.8 \( f + g(x) = x^3 + 3x^2 + x - 1 \), \( f - g(x) = x^3 - 3x^2 + x + 1 \), \( f \cdot g(x) = (x^3 + x)(3x^2 - 1) \) and \( f/g(x) = \frac{x^3+x}{3x^2-1} \)

1.9 a. \( f \circ g(x) = 2(3x+2)^2 - (3x+2) \), \( g \circ f(x) = 3(2x^2 - x) + 2 \), \( f \circ f(x) = 2(2x^2 - x) - 2(x^2 - x) \) and \( g \circ g(x) = 9x + 8 \) (D = \( (-\infty, \infty) \) for all) b. \( f \circ g(x) = \sqrt{x^2 - 1} \) (D = \( [1, \infty) \)), \( g \circ f(x) = x^2 - 1 \) (D = \( [1, \infty) \)), \( f \circ f(x) = \sqrt{x-1} - 1 \) (D = \( [2, \infty) \)) c. \( f \circ g(x) = \frac{1}{x^2+2x} \) (D = \( [x | x \neq 0) \)), \( g \circ f(x) = x^3 + x^1 \) (D = \( [x | x \neq 0) \)), \( f \circ f(x) = x = x \) (D = \( [x | x \neq 0) \) and \( g \circ g(x) = (x^3 + x)^3 + (x^3 + x) \) (D = \( R \))

1.10 a. \( f \circ f(x) = a^x \), D = \( (-\infty, \infty) \), R = \( (0, \infty) \) (the graph is increasing for \( a > 1 \), \( y = 1 \) for \( a = 1 \) and decreasing for \( 0 < a < 1 \))

1.11 a. Graph \( y = 2^x \) (it’s an increasing graph) and shift it 1 unit upward b. \( y = 3^{-x} = (\frac{1}{3})^x \) (decreasing graph) c. Graph \( y = 2^x \), reflect it about the x-axis and shift it 3 units upward (decreasing graph, y-intercept = 2)

1.12 a. \( a = 2, C = 3 \)

1.13 a. 6 b. -2 c. 15 d. 8

1.14 a. \( \ln(8) = 3 \ln(2) \) b. \( \ln(2^y) \)

1.15 a. \( 4 \ln(2) \) b. \( \frac{1}{e} \) c. \( 5 \log(3) = 5 \ln(3)/\ln(2) \)

1.16 a. \( \frac{e^x}{x-1} \) (x > 1 because of the domain of \( \ln(x-1) \) !)

1.17 a. If \( x_1 \neq x_2 \) implies \( f(x_1) \neq f(x_2) \) b. Every horizontal line intersects the graph at most once.

1.18 a. Yes b. No

1.19 a. \( f^{-1}(x) = \frac{5x-1}{2x+3} \) b. \( f^{-1}(x) = \frac{x^2-2}{5} \) (x ≥ 0) c. \( f^{-1}(x) = e^x - 3 \)

1.20 a. \( f^{-1}(n) = 3\ln(n/100) = 3\ln(n/100)/\ln(2) \) b. After about 26.9 hours (\( f^{-1}(500000) \))