Homework Assignment 2 Calculus I

a. If \( f(x) = \frac{x^2-9x+14}{4x-2x^2} \), evaluate the limit if it exists:

1. **[direct substitution]** \( \lim_{x \to 1} f(x) = \frac{1-9+14}{4-2} = 3 \)

2. **[form \( \frac{0}{0} \), use factoring]** \( \lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{(x-2)(x-7)}{2x(2-x)} = \lim_{x \to 2} \frac{x-7}{2x(2-x)} = \frac{7}{4} = \frac{5}{2} \)
   
   Note: we can cancel \( x-2 \) since in the denominator \( 2-x = -1 \times (x-2) \)

3. **[form \( \frac{\infty}{\infty} \), divide by \( x^2 \]** \( \lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{\frac{1}{x^2} + \frac{14}{x^2}}{\frac{4}{x^2} - \frac{2x}{x^2}} = \lim_{x \to \infty} \frac{1}{4} = \frac{1}{2} \)

4. **[form \( \frac{c\neq0}{0} \), use factoring and reasoning]** \( \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{(x-2)(x-7)}{2x(2-x)} = \lim_{x \to 0} \frac{x-7}{2x(2-x)} = \infty \)
   
   (if \( x \downarrow 0 \), then \( x \) is small positive and \( x-7 \) is negative, so \( \frac{x}{2-x} \) will become large positive.)

b. How would you define \( f(2) \) in a. to make \( f \) continuous at 2?  
   Answer: define \( f(2) = \frac{5}{4} \), the limiting value in exc. a.2. For other values \( f \) remains as defined.

c. Evaluate the limit if it exists:

1. \( \lim_{x \to 4} \frac{x^2-16}{\sqrt{x}-2} \) \( \left[ \text{form } \frac{0}{0}, \text{ use root trick} \right] \)

\[
\lim_{x \to 4} \frac{x^2-16}{\sqrt{x}-2} = \lim_{x \to 4} \frac{(x-4)(x+4)(\sqrt{x}+2)}{x-4} = \lim_{x \to 4} (x+4)(\sqrt{x}+2) = 32
\]

2. \( \lim_{x \to \infty} \left(\frac{1}{x} \right) = \lim_{x \to \infty} (4-1)^{-x} = \lim_{x \to \infty} 4^x = \infty \)

3. **[direct substitution]** \( \lim_{x \to 0} \frac{7+x}{\sqrt{7x}+2} = \lim_{x \to 0} \frac{(7+x)^{-1/2}}{2} = \lim_{x \to 0} \frac{1}{7} = 0 \)

4. **[form \( \frac{0}{0} \), use common (math) sense]**

\[
\lim_{x \to 0} \frac{7+x}{\sqrt{7x}+2} = \lim_{x \to 0} \frac{7+x}{x+2} = \lim_{x \to 0} \frac{7}{x+2} = \lim_{x \to 0} \frac{7}{2} = \frac{1}{49}
\]

d. If \( F(x) = \frac{x^2-4x}{|x-4|} \), evaluate the following limits if they exist:

Use \(|x-4| = \begin{cases} x-4, & \text{if } x \geq 4 \\ -(x-4), & \text{if } x < 4 \end{cases} \) so \( F(x) = \begin{cases} x^2-4x & \text{if } x \geq 4 \\ x^2-4x & \text{if } x < 4 \end{cases} \)

1. \( \lim_{x \to 4} F(x) = \lim_{x \to 4} \frac{x^2-4x}{x-4} = \lim_{x \to 4} \frac{x(x-4)}{x-4} = \lim_{x \to 4} x = 4 \)

2. \( \lim_{x \to 4} F(x) = \lim_{x \to 4} x^2-4x = \lim_{x \to 4} x^2 - 4x = \lim_{x \to 4} (x-4) = 4 \)

3. \( \lim_{x \to 4} F(x) \) d.n.e., since \( \lim_{x \to 4} F(x) \neq \lim_{x \to 4} F(x) \)

e. Find the vertical and horizontal asymptotes of the function \( f(x) = \frac{x^2-10x+25}{2x^2-4x-30} \)

Vertical asymptotes: if \( f(x) = \frac{(x-5)^2}{2(x-5)(x+3)} \) has shape \( \frac{c\neq0}{0} \) only if \( x = -3 \): \( \text{VA } x = -3 \)

HA: \( y = \frac{1}{2} \), since \( \lim_{x \to \infty} \frac{x^2-10x+25}{2x^2-4x-30} = \lim_{x \to \infty} \frac{\frac{1}{2} - \frac{5x}{x} + \frac{25}{x^2}}{2 - \frac{4}{x} - \frac{30}{x^2}} = 1 \) and

\[
\lim_{x \to \infty} \frac{x^2-10x+25}{2x^2-4x-30} = \frac{1}{2}
\]