An Immersed Boundary Method for Computing Anisotropic Permeability of Structured Porous Media

David Lopez Penha
with
Bernard Geurts, Steffen Stolz* and Markus Nordlund*

*Philip Morris Products S.A., PMI Research & Development, Neuchâtel, Switzerland

PhD-TW Colloquium
June 11, 2009
1. Averaged transport in porous media
2. Numerical flow predictions
3. Predicting permeability of structured porous media
4. Concluding remarks & outlook
Outline

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3. Predicting permeability of structured porous media
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source: http://gubbins.ncsu.edu/research.html

- Amorphous nano-porous material (e.g. porous glass)
Porous Media

... the household sponge ...

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Transport in Porous Media

Microscopic approach:

- Modeling flow at pore level
- Complex description of solid surfaces → body-fitted grid not available
- Real system measurements are impossible

Instead: coarsening of flow description (macroscopic approach)

- Technique: average variables over representative volume
- Avoid need for exact interphase boundaries
- Computationally much less demanding
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Microscopic Approach to Flow in Porous Media

Macroscopic approach: volume-averaged Navier-Stokes equations
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Transport Coefficients in Macroscopic Approach

Volume averaging of fluid variables

- Transport coefficient: generalized Darcy’s law (1856)

\[ \langle u_f \rangle = - \frac{k}{\mu_f} \cdot \nabla \langle p_f \rangle \]  
\[ \text{ (k: permeability tensor) } \]
Volume averaging of fluid variables

- Transport coefficient: generalized Darcy’s law (1856)

\[
\langle \mathbf{u}_f \rangle = -\frac{k}{\mu_f} \cdot \nabla \langle p_f \rangle \quad (k: \text{permeability tensor})
\]
Closure for the Permeability

Generalized Darcy’s law:

\[ \langle u_f \rangle = -\frac{k}{\mu_f} \cdot \nabla \langle p_f \rangle \]  

(k: permeability tensor)

- \textbf{k}: measure of fluid resistance
- Dependent on geometric features solid matrix
- Poses closing problem \( \longrightarrow \) predict numerically on representative volume
Closure for the Permeability

- **Generalized Darcy’s law:**

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Closure for the Permeability

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Immersed Boundary Technique

Navier-Stokes equations:

\[ \nabla \cdot \mathbf{u} = 0 \]
\[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \mathbf{f} \]

- \( \mathbf{f} \): forcing function \( \rightarrow \) no-slip condition
- Volume penalization:
  \[ \mathbf{f} = -\frac{1}{\epsilon} \mathcal{H}(\mathbf{u} - \mathbf{u}_s), \quad \epsilon \ll 1 \]
  - \( \mathcal{H} \): mask function \( \rightarrow \mathcal{H} = 1 \) inside solid, \( \mathcal{H} = 0 \) elsewhere
Immersed Boundary Technique

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\[ \diamond \mathbf{f}: \text{forcing function} \rightarrow \text{no-slip condition} \]

\[ \diamond \text{Volume penalization:} \]

\[ \mathbf{f} = -\frac{1}{\epsilon} H(\mathbf{u} - \mathbf{u}_s), \quad \epsilon \ll 1 \]

\[ \diamond H: \text{mask function} \rightarrow H = 1 \text{ inside solid, } H = 0 \text{ elsewhere} \]
Immersed Boundary Technique

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Structured Porous Medium

- **Representative volume:** spatially periodic array of staggered squares
- **Geometry:** Kuwahara et al., Int. J. Heat Mass Transfer 44, (2001)
Structured Porous Medium

◇ Velocity vector field at Re = 1
Structured Porous Medium

 veloc vector field at Re = 100
An Immersed Boundary Method for Computing Anisotropic Permeability of Structured Porous Media
Predicting Permeability in One Direction

- Representative volume
- Darcy's law: flow rate $\propto$ hydraulic jump
  \[
  \frac{Q}{A} = -\frac{k \Delta p}{\mu L}
  \]
- $k$: component of permeability tensor $k$
Predicting Permeability in One Direction

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- Darcy’s law: flow rate $\propto$ hydraulic jump

$$\frac{Q}{A} = -\frac{k}{\mu} \frac{\Delta p}{L}$$

- $k$: component of permeability tensor $\mathbf{k}$
Spatially Periodic Array of Cylinders

- 2D geometry considered by Edwards et al. (1990)
- Solution technique: FEM & body-fitted grid
Numerical Prediction of Permeability

- \( k \) (x-direction) vs. \( Re \)
- \( k \) (y-direction) vs. \( Re \)

- \( \{x, y\}\)-permeability vs. Reynolds number (various solidity)

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Conclusions & Outlook

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◊ Developed IB method for flow in "structured" porous media
◊ Applied to Kuwahara geometry - verified correct capturing of flow
◊ Prediction of permeability through Darcy's law

Outlook:

◊ Perform full parameter study of model porous media
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