Homework III, Introduction to Mathematical Systems Theory, 2001/2002

Hand out: February 18th

Hand in: March 5th

- Corporation in groups of two people of approximately the same level is allowed.
- Everybody hands in his or her *own* version.
- It is not allowed to copy from each other.

The numbers refer to the book.

1. Exercise 6.3. Remark: In the right hand side of the formula in Part (b), the term $\bar{q}_1$ should be replaced by q_1.
2. Exercise 7.21. Hint Determine a symmetric matrix $P = P^T$ such that $A^T P + PA = -I$. Show that P is positive definite.
3. Exercise 10.17.
4. Consider the system

$$\frac{d}{dt}x = Ax + Bu \qquad y = Cx$$

With:

$$A = \begin{pmatrix} 16 & 3 & 9 & -6 \\ 15 & 1 & 10 & -6 \\ 36 & 6 & 23 & -15 \\ 100 & 17 & 61 & -40 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -1 \\ 0 & 0 \\ 1 & 1 \\ 4 & -1 \end{pmatrix} \qquad C = \begin{pmatrix} 15 & 1 & 10 & -6 \end{pmatrix}$$

   (a) Is this system controllable?
   (b) Is this system observable?
   (c) Denote the columns of B by b_1 and b_2 respectively. Is (A, b_i) controllable for $i = 1$ or $i = 2$?
   (d) Determine a matrix N' such that $(A + BN', b_1)$ is controllable.
   (e) Determine the characteristic polynomial and the eigenvalues of A.
   (f) Is the system (asymptotically) stable?
   (g) Determine a state feedback $u = N_1 x$ that shifts as few poles as possible and such that the closed-loop system is just stable (not asymptotically stable).
   (h) Determine a state feedback $u = N_2 x$ that shifts all poles to -1.
   (i) Determine a matrix P such that $(A + BN_2)^T P + P(A + BN_2) = -I$. Is P positive definite?
   (j) Design an observer with observer poles in $-1, -2, -3 + i, -3 - i$.
   (k) Use the separation principle, the certainty equivalence priniciple, and the observer of Part 4j to approximate the feedback of Part 4h in the case that only the output can be measured for feedback.

   Provide the equations of the controlled system, i.e., the equations of the system, the observer and the controller. What are the poles of scontrolled this system?