An extended test coverage framework

*From potential to actual coverage*

Mark Timmer

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## Motivation for research on testing

- Software is getting more and more complex
- Bugs cost a lot of money
- Testing is a large part of software development
Introduction

Motivation for research on testing
- Software is getting more and more complex
- Bugs cost a lot of money
- Testing is a large part of software development

Motivation for research on test coverage
- Testing is inherently incomplete
- A notion of *quality* of a test suite is necessary
Motivation for research on testing

- Software is getting more and more complex
- Bugs cost a lot of money
- Testing is a large part of software development

Motivation for research on test coverage

- Testing is inherently incomplete
- A notion of quality of a test suite is necessary

Motivation for my research project

- Previous work by Laura Brandán Briones, Marielle and Ed
- Ideas for several improvements
Preliminaries – labeled transition systems

Definition LTSs

LTS \( A = \langle S, s^0, L, \Delta \rangle \), such that

- \( S \): set of states
- \( s^0 \): initial state
- \( L \): set of actions (partitioned into input actions and output actions)
- \( \Delta \): transition relation (assumed deterministic)
Definition LTSs

LTS $\mathcal{A} = \langle S, s^0, L, \Delta \rangle$, such that

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- $s^0$: initial state
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- $\Delta$: transition relation (assumed deterministic)
Preliminaries – test cases for LTSs

Perform an input
Observe all outputs
Always stop after an error

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Perform an input
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Perform an input
Preliminaries – test cases for LTSs

- Perform an input
- Observe all outputs
Preliminaries – test cases for LTSs

- Perform an input
- Observe all outputs
- Always stop after an error

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Preliminaries – Weighted fault models (WFM}s)

Restriction on weighted fault models

\[
0 < \sum_{\sigma \in L^*} f(\sigma) < \infty
\]
Preliminaries – Weighted fault models (WFMs)

\[
f(\text{coffee!}) = 10
\]

Restriction on weighted fault models

\[0 < \sum_{\sigma \in \mathcal{L}^*} f(\sigma) < \infty\]
Preliminaries – Weighted fault models (WFMs)

\[ f(\text{coffee!}) = 10 \]
\[ f(10\text{ct? tea!}) = 0 \]

Restriction on weighted fault models

\[ 0 < \sum_{\sigma \in L^*} f(\sigma) < \infty \]
Preliminaries – Weighted fault models (WFM)

Weighted fault models (WFMs) are a formalism for modeling the behavior of systems where the transitions between states are associated with weights. These weights can represent various properties, such as cost or reliability. The diagram illustrates a simple WFM with three states: $s_1$, $s_0$, and $s_2$. The states are connected by transitions labeled with "20ct?", "10ct?", and "δ", which stand for "20 cents?", "10 cents?", and "delta", respectively. The weight function $f$ assigns values to the transitions, indicating the cost or another measure associated with each transition. For example:

- $f(coffee!) = 10$
- $f(10ct? tea!) = 0$
- $f(10ct? coffee!) = 5$

These values can be used to evaluate the overall cost or weight of a particular sequence of transitions within the system.

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Preliminaries – Weighted fault models (WFM)

Restriction on weighted fault models

0 < \sum_{\sigma \in L^*} f(\sigma) < \infty

f(coffee!) = 10
f(10ct? tea!) = 0
f(10ct? coffee!) = 5
f(10ct? tea! 10ct? coffee!) = 3
Preliminaries – Weighted fault models (WFMs)

Restriction on weighted fault models

\[ 0 < \sum_{\sigma \in L^*} f(\sigma) < \infty \]

\[
\begin{align*}
 f(\text{coffee!}) &= 10 \\
 f(10\text{ct? tea!}) &= 0 \\
 f(10\text{ct? coffee!}) &= 5 \\
 f(10\text{ct? tea! 10ct? coffee!}) &= 3
\end{align*}
\]
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Preliminaries – Weighted fault models (WFMs)

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Assume \[ \sum_{\sigma \in L^*} f(\sigma) = 150 \]

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Assume \( \sum_{\sigma \in L^*} f(\sigma) = 150 \)

\( tot\text{Cov}_p = 150 \)
Assume $\sum_{\sigma \in L^*} f(\sigma) = 150$

$totCov_p = 150$

$absCov_p = 7 + 4 + 6 + 9 + 2 = 28$
Assume $\sum_{\sigma \in L^*} f(\sigma) = 150$

$totCov_p = 150$

$absCov_p = 7 + 4 + 6 + 9 + 2 = 28$

$relCov_p = \frac{28}{150} = 0.19$
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Preliminaries – Weighted fault models (WFMAs)

\[ \text{absCov}_p = 7 + 4 + 6 + 9 + 2 + 11 + 6 = 45 \]
Definition of fault automata (FAs)

Fault automaton: an LTS and a function \( r \) assigning these weights.

We require that \( r(s, a) = 0 \) for correct outputs.

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**Preliminaries - Fault automata**

**Definition of fault automata (FAs)**

Fault automaton: an LTS and a function $r$ assigning these weights. We require that $r(s, a) = 0$ for correct outputs.
Definition of fault automata (FAs)

Fault automaton: an LTS and a function $r$ assigning these weights. We require that $r(s, a!) = 0$ for correct outputs.
Problem: infinite traces over FA, so \( \sum_{\sigma \in L^*} f(\sigma) \neq \infty \)
From fault automaton to weighted fault model

Problem: infinite traces over FA, so $\sum_{\sigma \in L^*} f(\sigma) \not< \infty$

Solutions:
- Discard traces with length larger than some threshold
- Discount error weights by their depth
Problem: infinite traces over FA, so $\sum_{\sigma \in L^*} f(\sigma) \not< \infty$

Solutions:
- Discard traces with length larger than some threshold
- Discount error weights by their *depth*

Not relevant for my work.
Limitations of potential coverage

Previous work on potential coverage:

\[ \text{absCov}(f,t) = 28 \]

**Limitations of potential coverage**

Errors that are potentially covered

All these errors are not actually covered in every execution

What if the test case is executed multiple times?

**Actual coverage**

What is actually covered

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Limitations of potential coverage

Previous work on potential coverage: 
\[ \text{absCov}_p(f, t) = 28 \]
Limitations of potential coverage

Previous work on potential coverage:
\( \text{absCov}_p(f, t) = 28 \)

Limitations of potential coverage

- Errors that are \textit{potentially} covered
Limitations of potential coverage

Previous work on potential coverage:
\( absCov_p(f, t) = 28 \)

- Errors that are *potentially* covered
- All these errors are not actually covered in every execution
Limitations of potential coverage

Previous work on potential coverage:
\[ \text{absCov}_p(f, t) = 28 \]

Limitations of potential coverage

- Errors that are potentially covered
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- What if the test case is executed multiple times?
Limitations of potential coverage

Previous work on potential coverage: $\text{absCov}_p(f, t) = 28$

Limitations of potential coverage

- Errors that are potentially covered
- All these errors are not actually covered in every execution
- What if the test case is executed multiple times?

Actual coverage

- What is actually covered
Limitations of potential coverage

Actual coverage

- Execution coverage:
  Faults covered when observing a specific execution

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Limitations of potential coverage

Actual coverage

- Execution coverage: Faults covered when observing a specific execution
- Actual coverage: Probability mass distribution expressing execution coverage of single or sequence of executions
Limitations of potential coverage

Actual coverage

- Execution coverage: Faults covered when observing a specific execution
- Actual coverage: Probability mass distribution expressing execution coverage of single or sequence of executions
- Expected actual coverage

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Requirements for actual coverage

Actual coverage
Probability mass distribution expressing execution coverage of single or sequence of executions

- Indication of confidence in our knowledge on error presence
Requirements for actual coverage

Actual coverage

Probability mass distribution expressing execution coverage of single or sequence of executions

- Indication of confidence in our knowledge on error presence
- Include number of executions. More executions, more coverage
Requirements for actual coverage

Actual coverage

Probability mass distribution expressing execution coverage of single or sequence of executions

- Indication of confidence in our knowledge on error presence
- Include number of executions. More executions, more coverage
- For \( n \to \infty \) executions, equal to potential coverage
Requirements for actual coverage

**Actual coverage**

Probability mass distribution expressing execution coverage of single or sequence of executions

- Indication of confidence in our knowledge on error presence
- Include number of executions. More executions, more coverage
- For $n \to \infty$ executions, equal to potential coverage
- Observing an error: total coverage
Actual coverage

Probability mass distribution expressing execution coverage of single or sequence of executions

- Indication of confidence in our knowledge on error presence
- Include number of executions. More executions, more coverage
  
  For \( n \to \infty \) executions, equal to potential coverage
- Observing an error: total coverage
- *Not* observing an error: increase of coverage, yet no total coverage
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Motivation for the model

Actual coverage:
Which errors will actually be covered?
Motivation for the model

Actual coverage:
Which errors will actually be covered?

Necessary:
- Probabilistic transition behaviour
Motivation for the model

Actual coverage:
Which errors will actually be covered?

Necessary:
- Probabilistic transition behaviour
- Occurrence probabilities

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Motivation for the model

Actual coverage:
Which errors will actually be covered?

Necessary:
- Probabilistic transition behaviour
- Occurrence probabilities

Approach:
- Probabilities of correct outputs
- Probabilities of the presence of errors
- Probabilities of the occurrence of erroneous behaviour
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Definition of the correctness probability function

Correctness probability function:
- 0 for incorrect outputs
- 0 for transitions not included in the test case

Values known from implementation or measured.
Probabilities of the presence and occurrence of errors

**Fault presence function**
Gives the probability that a certain error is made

**Error occurrence function**
Gives the probability that a certain error occurs, *given its presence*
Probabilistic transition behaviour

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Probabilistic transition behaviour

\[ p = p_f \times p_o \]
Probabilistic transition behaviour

Erroneous outputs:
\[ p = p_f \times p_o \]
Probabilistic transition behaviour

Erroneous outputs:
\[ p = p_f \times p_o \]

Correct outputs:
\[ p = p_c \times (1 - \sum p_{error}) \]
Probabilistic transition behaviour

Erroneous outputs:
\[ p = p_f \times p_o \]

Correct outputs:
\[ p = p_c \times (1 - \sum p_{error}) \]

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Path probabilities

\[ p(a? \ e! \ b?)(d!) = 0.025 \]
Path probabilities

\[ p(a? \ e! \ b?)(d!) = 0.025 \]
Path probabilities

\[
p(a? \ e! \ b?)(d!) = 0.025
\]

\[
\bar{p}(a? \ e! \ b? \ d!) = 1.0 \cdot 0.2475 \cdot 1.0 \cdot 0.025 = 0.006
\]
An execution covers an error if it passes it. Coverage fraction: the confidence in our knowledge. Observing an error yields total certainty: $\text{CovFrac} = 1$. Not observing an error $n$ times: $\text{CovFrac} = 1 - (1 - p)^n$. 

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An execution *covers* an error if it passes it.
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An execution *covers* an error if it passes it.

*Coverage fraction*: the confidence in our knowledge.
An execution *covers* an error if it passes it.

*Coverage fraction*: the confidence in our knowledge.

Observing an error yields total certainty: $\text{CovFrac} = 1$. 

[Diagram of a tree structure with nodes labeled as 'fail' or 'pass' and probabilities indicated.]

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An execution covers an error if it passes it.

- **Coverage fraction**: the confidence in our knowledge.
- Observing an error yields total certainty: \( \text{CovFrac} = 1 \).
- Not observing an error \( n \) times: \( \text{CovFrac} = 1 - (1 - p_o)^n \)
Execution coverage

Def. of execution coverage

\[ \text{absExCov}(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \text{CovFrac}(\sigma', p_o, \sigma) \]
Execution coverage

\[ \text{Def. of execution coverage} \]

\[ \text{absExCov}(\sigma, t, f, p_\sigma) = \sum_{\sigma' \in t} f(\sigma') \cdot \text{CovFrac}(\sigma', p_\sigma, \sigma) \]

\[ \text{absExCov}(..) = \]

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Execution coverage

\[ \text{absExCov}(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \text{CovFrac}(\sigma', p_o, \sigma) \]

absExCov(..) =
Execution coverage

Def. of execution coverage

\[
\text{absExCov}(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \text{CovFrac}(\sigma', p_o, \sigma)
\]

absExCov(..) = 7 \cdot (1 - (1 - 0.2)^1) +
Def. of execution coverage

$$absExCov(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \text{CovFrac}(\sigma', p_o, \sigma)$$

$$absExCov(\ldots) = 7 \cdot (1 - (1 - 0.2)^1) +$$
Def. of execution coverage

\[
\text{absExCov}(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \text{CovFrac}(\sigma', p_o, \sigma)
\]

absExCov(\ldots) = 7 \cdot (1 - (1 - 0.2)^1) + 4 \cdot 0.5 + 6 \cdot 0.8 = 8.2
Def. of execution coverage

\[
\text{absExCov}(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \text{CovFrac}(\sigma', p_o, \sigma)
\]

\[
\text{absExCov}(..) = 7 \cdot (1 - (1 - 0.2)^1) + 4 \cdot 0.5 + 6 \cdot 0.8 = 8.2
\]

For three times this execution:
\[
\text{absExCov}(..) = 7 \cdot (1 - (1 - 0.2)^3) + \cdots = 12.868
\]
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Actual coverage of test cases

The actual coverage of a single execution is a random variable.

\[ P[absCov_{t,f,p,o}^{\text{single}} = x] = \sum_{\sigma \in \text{exec}_t} \bar{p}(\sigma) \]

\[ \text{absExCov}(\sigma, t, f, p_o) = x \]
Actual coverage of test cases

The actual coverage of a sequence of execution is also a random variable.

\[ \Pr[\text{absCov}^n_{t,f,p,o} = x] = \sum_{E \in \text{exec}^n_t} \bar{p}(E) \]

where \( \text{absExCov}(E,t,f,p,o) = x \)
Expected actual coverage

\[ E(\text{absCov}_{t,f,p,p_0}^{\text{single}}) = \sum_{\sigma \in \text{exec}} \text{absExCov}(\sigma, t, f, p_0) \cdot \bar{p}(\sigma) \]
Expected actual coverage

\[ E(\text{absCov}_{t,f,p,p_o}^{\text{single}}) = \sum_{\sigma \in \text{exec}_t} \text{absExCov}(\sigma, t, f, p_o) \cdot \bar{p}(\sigma) \]

Potential coverage

Absolute potential coverage: 28
\[ E(\text{absCov}^{\text{single}}_{t,f,p,p_o}) = \sum_{\sigma \in \text{exec}_t} \text{absExCov}(\sigma, t, f, p_o) \cdot \overline{p}(\sigma) \]

Potential coverage

Absolute potential coverage: 28

Actual coverage

\[ E(\text{absCov}^{\text{single}}_{t,f,p,p_o}) = \text{absExCov}(a? e! b? d!, t, f, p_o) \cdot \overline{p}(a? e! b? d!) + \text{absExCov}(a? e! b? e!, t, f, p_o) \cdot \overline{p}(a? e! b? e!) + \text{absExCov}(a? e! b? c!, t, f, p_o) \cdot \overline{p}(a? e! b? c!) + \cdots + \text{absExCov}(a? c!, t, f, p_o) \cdot \overline{p}(a? c!) \]
**Expected actual coverage**

\[
E(\text{absCov}^{\text{single}}_{t,f,p,p_0}) = \sum_{\sigma \in \text{exec}} \text{absExCov}(\sigma, t, f, p_0) \cdot \bar{p}(\sigma)
\]

---

**Potential coverage**

**Absolute potential coverage: 28**

**Actual coverage**

\[
E(\text{absCov}^{\text{single}}_{t,f,p,p_0}) =
\text{absExCov}(a? e! b? d!, t, f, p_0) \cdot \bar{p}(a? e! b? d!)+
\text{absExCov}(a? e! b? e!, t, f, p_0) \cdot \bar{p}(a? e! b? e!)+
\text{absExCov}(a? e! b? c!, t, f, p_0) \cdot \bar{p}(a? e! b? c!)+
\cdots + \text{absExCov}(a? c!, t, f, p_0) \cdot \bar{p}(a? c!)
\]
Expected actual coverage

\[ E(\text{absCov}_{t,f,p,p_0}^{\text{single}}) = \sum_{\sigma \in \text{exec}} \text{absExCov}(\sigma, t, f, p_o) \cdot \bar{p}(\sigma) \]

Potential coverage

Absolute potential coverage: 28

Actual coverage

\[ E(\text{absCov}_{t,f,p,p_0}^{\text{single}}) = (7 \cdot 0.2 + 4 \cdot 1 + 6 \cdot 0.8) \cdot \bar{p}(a? e! b? d!) + \text{absExCov}(a? e! b? e!, t, f, p_o) \cdot \bar{p}(a? e! b? e!) + \text{absExCov}(a? e! b? c!, t, f, p_o) \cdot \bar{p}(a? e! b? e!) + \cdots + \text{absExCov}(a? c!, t, f, p_o) \cdot \bar{p}(a? c!) \]
Expected actual coverage

$$E(\text{absCov}_{t,f,p,p_o}^{\text{single}}) = \sum_{\sigma \in \text{exec}_t} \text{absExCov}(\sigma, t, f, p_o) \cdot \bar{p}(\sigma)$$

Potential coverage

Absolute potential coverage: 28

Actual coverage

$$E(\text{absCov}_{t,f,p,p_o}^{\text{single}}) = (7 \cdot 0.2 + 4 \cdot 1 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.025) + \text{absExCov}(a? e! b? e!, t, f, p_o) \cdot \bar{p}(a? e! b? e!) + \text{absExCov}(a? e! b? c! , t, f, p_o) \cdot \bar{p}(a? e! b? c!) + \cdots + \text{absExCov}(a? c!, t, f, p_o) \cdot \bar{p}(a? c!)$$
Expected actual coverage

\[ E(\text{absCov}^{\text{single}}_{t,f,p,p_o}) = \sum_{\sigma \in \text{exec}_t} \text{absExCov}(\sigma, t, f, p_o) \cdot \bar{p}(\sigma) \]

Potential coverage

Absolute potential coverage: 28

Actual coverage

\[ E(\text{absCov}^{\text{single}}_{t,f,p,p_o}) = \\
(7 \cdot 0.2 + 4 \cdot 1 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.025) + \\
\text{absExCov}(a? e! b? e!, t, f, p_o) \cdot \bar{p}(a? e! b? e!) + \\
\text{absExCov}(a? e! b? c!, t, f, p_o) \cdot \bar{p}(a? e! b? c!) + \\
\cdots + \text{absExCov}(a? c!, t, f, p_o) \cdot \bar{p}(a? c!) \]
\[ E(\text{absCov}^{\text{single}}) = \sum_{\sigma \in \text{exec}} \text{absExCov}(\sigma, t, f, p_o) \cdot \bar{p}(\sigma) \]

Potential coverage

Absolute potential coverage: 28

Actual coverage

\[ E(\text{absCov}^{\text{single}}) = (7 \cdot 0.2 + 4 \cdot 1 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.025) + (7 \cdot 0.2 + 4 \cdot 0.5 + 6 \cdot 0.8) \cdot \bar{p}(a? \ e! \ b? \ e!) + \]
\[ \text{absExCov}(a? \ e! \ b? \ c!, t, f, p_o) \cdot \bar{p}(a? \ e! \ b? \ c!) + \]
\[ \cdots + \text{absExCov}(a? \ c!, t, f, p_o) \cdot \bar{p}(a? \ c!) \]
Expected actual coverage

\[ E(\text{absCov}_{t,f,p,p_o}^{\text{single}}) = \sum_{\sigma \in \text{exec}_t} \text{absExCov}(\sigma, t, f, p_o) \cdot \overline{p}(\sigma) \]

Potential coverage

Absolute potential coverage: 28

Actual coverage

\[ E(\text{absCov}_{t,f,p,p_o}^{\text{single}}) = \\
(7 \cdot 0.2 + 4 \cdot 1 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.025) + \\
(7 \cdot 0.2 + 4 \cdot 0.5 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.935) + \\
\text{absExCov}(a? e! b? c!, t, f, p_o) \cdot \overline{p}(a? e! b? c!)+ \\
\cdots + \text{absExCov}(a? c!, t, f, p_o) \cdot \overline{p}(a? c!) \]
Expected actual coverage

\[ E(\text{absCov}^{\text{single}}_{t,f,p,p_o}) = \sum_{\sigma \in \text{exec}} \text{absExCov}(\sigma, t, f, p_o) \cdot \bar{p}(\sigma) \]

Potential coverage

Absolute potential coverage: 28

Actual coverage

\[ E(\text{absCov}^{\text{single}}_{t,f,p,p_o}) = (7 \cdot 0.2 + 4 \cdot 1 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.025) + \\
(7 \cdot 0.2 + 4 \cdot 0.5 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.935) + \\
\text{absExCov}(a? e! b? c!, t, f, p_o) \cdot \bar{p}(a? e! b? c!) + \\
\cdots + \text{absExCov}(a? c!, t, f, p_o) \cdot \bar{p}(a? c!) = 8.3 \]
Expected actual coverage

Expected value of the actual coverage for a sequence of executions

\[ E(\text{absCov}_{t, f, p, p_o}^n) = \sum_{E \in \text{exec}_t^n} \text{absExCov}(E, t, f, p_o) \cdot \bar{p}(E) \]
Expected actual coverage

Expected value of the actual coverage for a sequence of executions

\[
E\left(\text{absCov}^n_{t,f,p,p_o}\right) = \sum_{E \in \text{exec}^n_t} \text{absExCov}(E, t, f, p_o) \cdot \bar{p}(E)
\]

Problem: exponential in \(n\), so not very feasible in practice.
I found a solution:

Expected value of the actual coverage for a sequence of executions

\[ E(\text{absCov}^n_{t,f,p,p_o}) = \]
\[ \sum_{\sigma a \in t} \left( f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^n \right) \cdot 1 + \right. \]
\[ \left. \sum_{i=0}^{n} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} (1 - p(\sigma,a))^i (1 - (1 - p_o(\sigma,a))^i) \right) \]
I found a solution:

**Expected value of the actual coverage for a sequence of executions**

\[
E(\text{absCov}^n_{t,f,p,p_o}) = \\
\sum_{\sigma a \in t} \left( f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^n \right) \cdot 1 + \\
\sum_{i=0}^{n} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} (1 - p(\sigma, a))^i (1 - (1 - p_o(\sigma, a))^i) \right) \right)
\]
I found a solution:

**Expected value of the actual coverage for a sequence of executions**

\[
E(\text{absCov}_{t,f,p,p_o}^n) = \\
\sum_{\sigma a \in t} \left( f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^n \right) \cdot 1 + \\
\sum_{i=0}^{n} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} (1 - p(\sigma, a))^i (1 - (1 - p_o(\sigma, a))^i) \right)
\]
I found a solution:

\[
E(\text{absCov}_{t,f,p,p_o}^n) = \\
\sum_{\sigma a \in t} \left( f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^n \right) \cdot 1 + \\
\sum_{i=0}^{n} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} (1 - p(\sigma, a))^i (1 - (1 - p_o(\sigma, a))^i) \right)
\]
I found a solution:

Expected value of the actual coverage for a sequence of executions

\[ E(\text{absCov}_{t,f,p,p_o}^n) = \sum_{\sigma a \in t} f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^n \right) \cdot 1 + \sum_{i=0}^{n} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} (1 - p(\sigma, a))^i (1 - (1 - p_o(\sigma, a))^i) \) \]
I found a solution:

Expected value of the actual coverage for a sequence of executions

\[
E(\text{absCov}_t,f,p,p_o) = \\
\sum_{\sigma a \in t} \left( f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^n \right) \cdot 1 + \\
\sum_{i=0}^{n} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} (1 - p(\sigma, a))^i (1 - (1 - p_o(\sigma, a))^i) \right)
\]
Example of actual coverage

Potential coverage
Absolute potential coverage: 28

Actual coverage

$E(\text{absCov}_{t,f,p,po}^5) =$

Actual coverage

$E(\text{absCov}_{t,f,p,po}^{\text{single}}) = 8.3$
Example of actual coverage

Potential coverage

Absolute potential coverage: 28

Actual coverage

\[
E(\text{absCov}_{t,f,p,p_o}^5) = 7 \cdot \left( (1 - (1 - 0.01)^5) \cdot 1 + \sum_{i=0}^{5} \binom{5}{i} 1^i \cdot 0.5^{5-i} \cdot (1 - 0.01)^i \cdot (1 - (1 - 0.2)i) \right)
\]

Actual coverage

\[
E(\text{absCov}_{t,f,p,p_o}^{\text{single}}) = 8.3
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Example of actual coverage

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\[ E(\text{absCov}^5_{t,f,p,p_0}) = 7 \cdot \left( (1 - (1 - 0.01)^5) \cdot 1 + \sum_{i=0}^{5} \left( \binom{5}{i} 1^i \cdot 0.01^{5-i} \cdot (1 - (1 - 0.2)^i) \right) \right) + 4 \cdot \left( (1 - (1 - 0.2475 \cdot 0.025)^5) \cdot 1 + \sum_{i=0}^{5} \left( \binom{5}{i} 0.2475^i \cdot (1 - 0.2475)^{5-i} \cdot (1 - 0.025)^i \cdot (1 - (1 - 0.5)^i) \right) \right) + \cdots \]
Example of actual coverage

Potential coverage

Absolute potential coverage: 28

Actual coverage

\[ E(\text{absCov}_t,f,p,p_o) = 8.3 \]

\[ = 21.45 \]
Asymptotical behaviour

Theorem

\[ \lim_{n \to \infty} E(\text{absCov}_t^n, f, p, p_0) = \text{absCov}_p(t, f) \]
Contents

Mark Timmer
An extended test coverage framework
Actual coverage of test suites

Mark Timmer  An extended test coverage framework
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\[ E(\text{absCov}_{t,f,p,p_o}^n) = \sum_{\sigma_a \in t} \left( f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^n \right) \cdot 1 + \sum_{i=0}^{n} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} (1 - p(\sigma, a)^i) (1 - (1 - p_o(\sigma, a))^i) \right) \]
Actual coverage of test suites

\[
E(\text{absCov}_{t,f,p,p_o}^n) = \sum_{\sigma a \in t} \left( f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^c(\sigma)^n \right) \right) 
\]

\[
+ \frac{c(\sigma)^n}{\sum_{i=0}^n \left( \binom{c(\sigma)^n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^c(\sigma)^{n-i} \right) (1 - p(\sigma, a)^i)(1 - (1 - p_o(\sigma, a)^i))}
\]

c(\sigma): the fraction of test cases that observe after \( \sigma \).
Conclusions

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